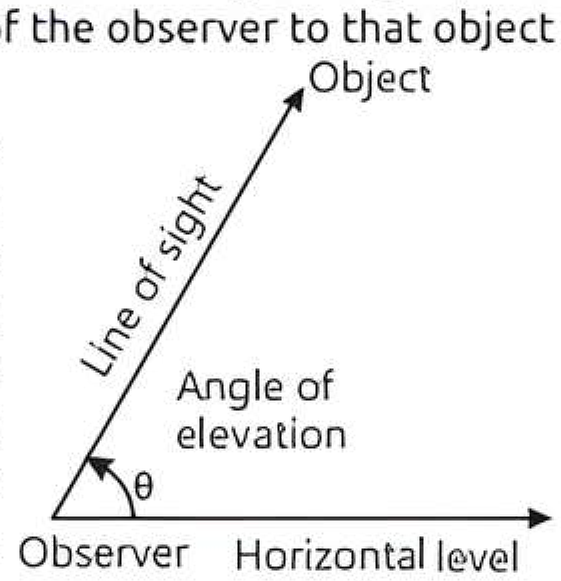
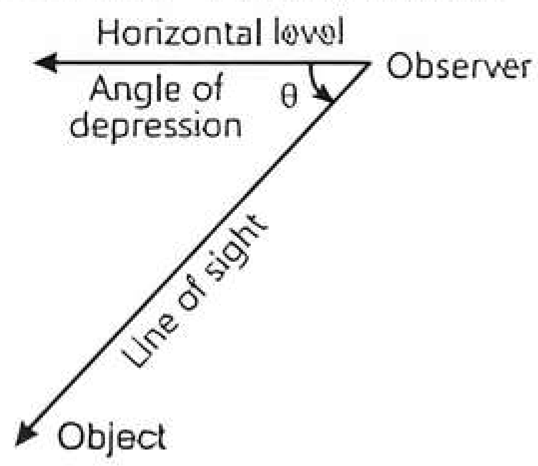
9 Some Applications of Trigonometry

Fastrack Revision

- **Line of Sight:** If an observer is viewing an object, the straight line joining the eye of the observer to that object is called line of sight.
- **Angle of Elevation:** The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, *i.e.*, the case when we raise our head to look at the object.



- **Angle of Depression:** The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, *i.e.*, the case when we lower our head to look at the object.



- The height or length of an object or the distance between two distant objects can be determined through trigonometric ratios.

Knowledge BOOSTER

1. *The angles of elevation and depression are always acute angles.*
2. *If the angle of elevation of the tower (or Sun) decreases, the shadow of the tower (or Sun) increases.*
3. *If the observer moves towards (or moves away) the perpendicular line, the angle of elevation increases (or decreases).*
4. *If the height of tower is doubled and the distance between the observer and the foot of tower is also doubled, then the angle of elevation remains same.*

Practice Exercise

Multiple Choice Questions

Q 1. If the height of the tower is equal to the length of its shadow, then the angle of elevation of the Sun is: *[CBSE SQP 2023-24]*

a. 30° b. 45° c. 60° d. 90°

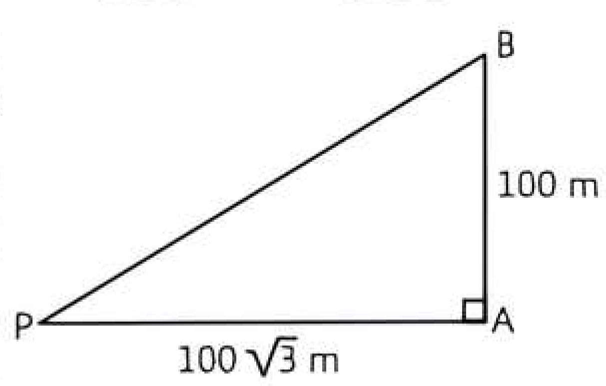
Q 2. The angle subtended by a tower of height 200 metres at a point 200 metres from the base is: *[CBSE 2023]*

a. 30° b. 45° c. 60° d. 0°

Q 3. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then Sun's elevation is: *[NCERT EXEMPLAR; CBSE 2023; CBSE SQP 2023-24]*

a. 60° b. 45° c. 30° d. 90°

Q 4. The angle subtended by a vertical pole of height 100 m at a point on the ground $100\sqrt{3}$ m from the base is, has measure of: *[CBSE 2023]*



a. 90° b. 60° c. 45° d. 30°

Q 5. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$, then the angle of elevation of the Sun is:

a. 45° b. 30° c. 60° d. 90°

Q 6. From a point P on a level ground, the angle of elevation of the top of tower is 30°. If the tower is 100 m high, the distance of point P from the foot of the tower is:

a. 149 m b. 156 m c. 173 m d. 200 m

Q 7. The angle of elevation of the top of the tower is 60° and the horizontal distance from the observer's eye to the foot of the tower is 100 m, then the height of the tower will be:

a. $50\sqrt{3}$ m b. $\frac{100}{\sqrt{3}}$ m

c. $100\sqrt{3}$ m d. $60\sqrt{3}$ m

Q 8. The string of a kite in air is 50 m long and it makes an angle of 60° with the horizontal. Assuming the string to be straight, the height of the kite from the ground is: *[CBSE 2023]*

a. $50\sqrt{3}$ m b. $\frac{100}{\sqrt{3}}$ m

c. $\frac{50}{\sqrt{3}}$ m d. $25\sqrt{3}$ m

Q 9. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is:

a. 3 m b. 6 m c. 8 m d. 9.2 m

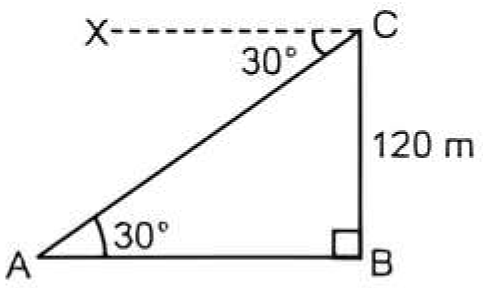
Q 10. A boy standing on top of a tower of height 20 m observes that the angle of depression of a car on the road is 60°. The distance between the foot of the tower and the car must be: [Use $\sqrt{3} = 1.73$]

a. 10.45 m b. 11.54 m c. 12.55 m d. 12.50 m

Q 11. If the angle of depression of an object from a 50 m high tower is 30°, then the distance of the object from the tower is:

a. $25\sqrt{3}$ m b. $\frac{50}{\sqrt{3}}$ m c. $50\sqrt{3}$ m d. 50 m

Q 12. The angle of depression of a car parked on the road from the top of 120 m high tower is 30°. The distance of the car from the tower (in metres) is:



a. $120\sqrt{3}$ m b. 120 m

c. $40\sqrt{3}$ m d. None of these

Q 13. A vertical straight tree of 15 m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break? [Use $\sqrt{3} = 1.73$]

a. 6.9 m b. 9.6 m c. 5.9 m d. 7.9 m

Q 14. An observer 1.6 m tall is 20 m away from a tower. The angle of elevation from his eye to the top of the tower is 45°. The height of the tower is:

a. 21.6 m b. 2 m

c. 72 m d. None of these

Assertion & Reason Type Questions

Directions (Q. Nos. 15-18): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 15. Assertion (A): If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the Sun is 45°.

Reason (R): Trigonometric ratio, tangent is defined as

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

Q 16. Assertion (A): The angle of elevation of the top of a tower is 60°. If the height of the tower and its base is tripled then angle of elevation of its top will also be tripled.

Reason (R): In an equilateral triangle of side $3\sqrt{3}$ cm, the length of the altitude is 4.5 cm.

Q 17. Assertion (A): Suppose a bird was sitting on a tree. A person was sitting on a ground and saw the bird, which makes an angle such that $\tan\theta = \frac{12}{5}$. The distance from bird to the person is 13 units.

Reason (R): In a right-angled triangle,

$$(\text{Hypotenuse})^2 = (\text{Side})^2 + (\text{Base})^2.$$

Q 18. Assertion (A): The angle of elevation of the top of the tower is 30° and the horizontal distance from the observer's eye to the foot of the tower is 50 m, then the height of the tower will be $\frac{50}{3}\sqrt{3}$ m.

Reason (R): While using the concept of angle of elevation/depression, triangle should be a right angled triangle.

Fill in the Blanks Type Questions

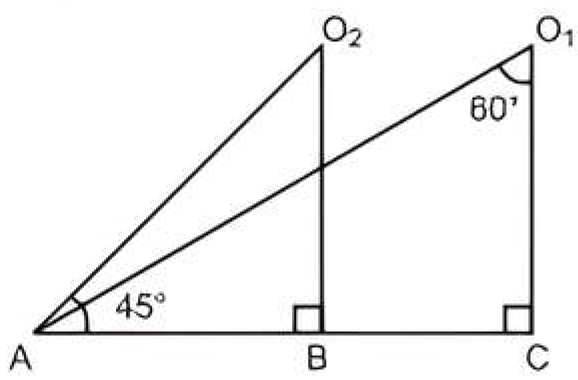
Q 19. The .... of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Q 20. The angle of .... of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level.

Q 21. If the length of the shadow of a tower is $\sqrt{3}$ times its height, the angle of elevation of the Sun is .... *[NCERT EXEMPLAR]*

Q 22. From a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°, then the height of the tower is .... [Use $\sqrt{3} = 1.73$]

Q 23. In the given figure, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are .... and ....



True/False Type Questions

Q 24. The height of the tower if length of the shadow is 10 m and Sun's altitude is 45°, is 10 m.

Q 25. If length of shadow of tower is 20 m and angle of elevation is 60°, then the height of tower is $\frac{20}{\sqrt{3}}$ m.

Q 26. A little boy is flying a kite. The string of kite makes an angle of 30° with the ground. If the height of the kite is 21m, then the length of the string is 35 m.

Q 27. The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled then angle of elevation of its top will also be doubled. *[NCERT EXERCISE]*

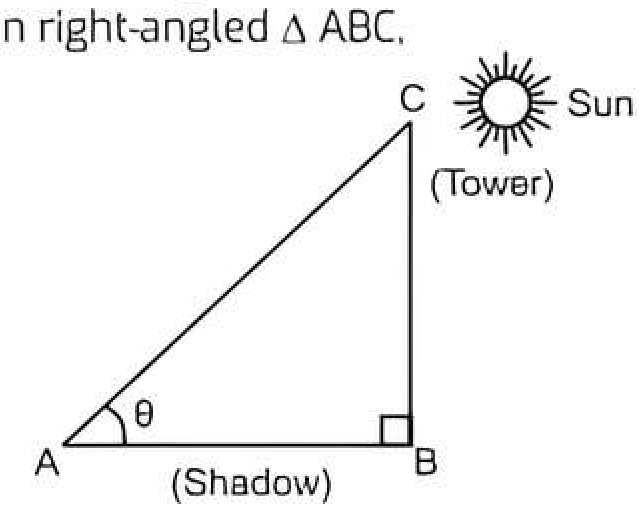
Solutions

1. (b) According to condition, the height of the tower is equal to the length of its Shadow, *i.e.*,

$$BC = AB \quad ...(1)$$

Let θ be the angle of elevation of the Sun.

Now In right-angled Δ ABC,



$$\tan\theta = \frac{BC}{AB} = \frac{AB}{AB} \quad \text{[from eq. (1)]}$$

$\Rightarrow \quad \tan\theta = 1$

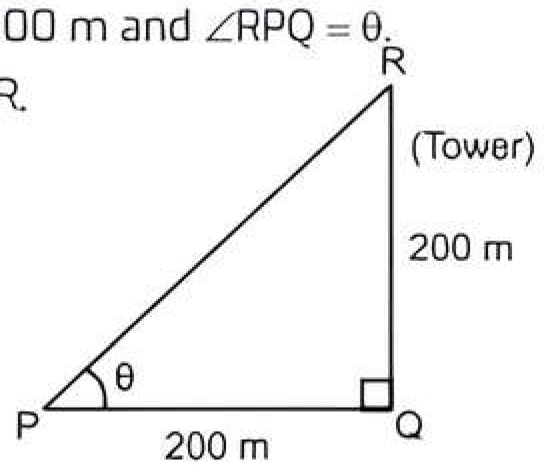
$\Rightarrow \quad \tan\theta = \tan 45°$

$\Rightarrow \quad \theta = 45°.$

2. (b) Let the angle subtended by a tower (RQ) at a point P from the base Q is θ.

Given, RQ = 200m, PQ = 200 m and ∠RPQ = θ.

Now In right-angled Δ PQR,



$$\tan\angle RPQ = \frac{RQ}{PQ}$$

$\Rightarrow \quad \tan\theta = \frac{200}{200} = 1$

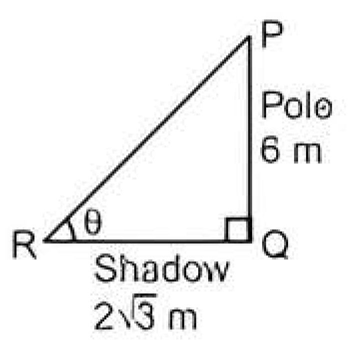
$\Rightarrow \quad \tan\theta = \tan 45°$

$\Rightarrow \quad \theta = 45°.$

3. (a) Let PQ = 6 m be the height of the pole and $RQ = 2\sqrt{3}$ m be its shadow.

Let angle of elevation of the Sun be θ.

In right-angled ΔPQR,



$$\tan\theta = \frac{PQ}{RQ} \quad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$= \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60°$$

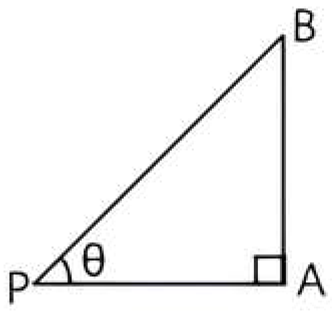
$\Rightarrow \quad \theta = 60°.$

4. (d) Given, height of the pole, AB = 100 m and distance of a point P from the base A, $AP = 100\sqrt{3}$m

Let θ be the angle Subtended by pole (AB) to the point P.

In right-angled Δ PAB,

$$\tan\theta = \frac{AB}{PA}$$



$\Rightarrow \quad \tan\theta = \frac{100}{100\sqrt{3}} = \frac{1}{\sqrt{3}}$

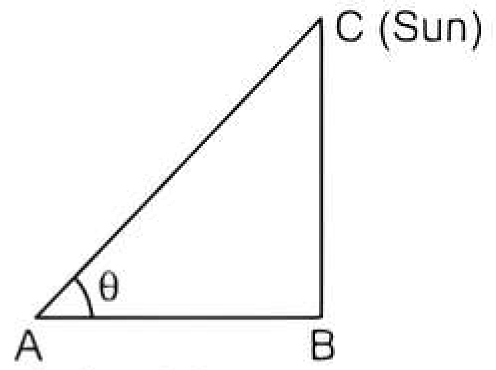
$\Rightarrow \quad \tan\theta = \tan 30°.$

$\Rightarrow \quad \theta = 30°.$

5. (b) Let C be the position of the Sun.

Let BC and AB be the length of rod and length of the shadow.

$$\text{Given, } \frac{\text{Length of rod}}{\text{Length of shadow}} = \frac{1}{\sqrt{3}} = \frac{BC}{AB} \quad ...(1)$$



In right-angled ΔABC,

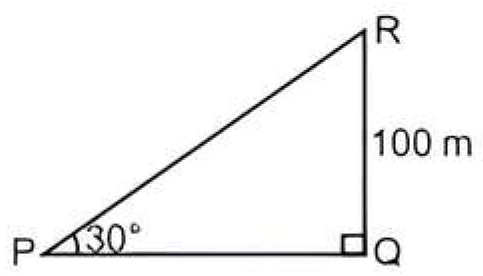
$$\tan\theta = \frac{BC}{AB}$$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad (\because \text{from eq. (1)})$

$\Rightarrow \quad \tan\theta = \tan 30°$

$\Rightarrow \quad \theta = 30°.$

6. (c) Let QR = 100 m be the height of the tower and point P makes an angle of elevation of the top of the tower, *i.e.*, ∠QPR = 30°.



In right-angled ΔPQR,

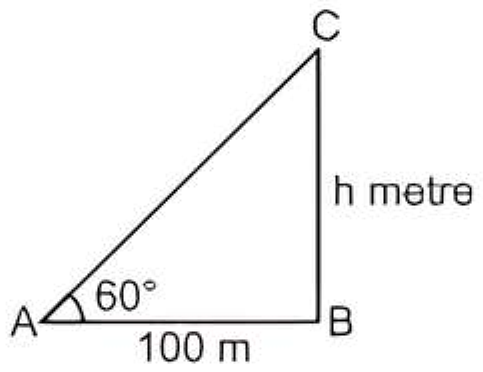
$$\tan 30° = \frac{RQ}{PQ} \quad \left(\because \tan\theta = \frac{P}{B}\right)$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{PQ}$

$\Rightarrow \quad PQ = 100\sqrt{3}$ m

$= 100 \times 1.73$ m $= 173$ m

7. (c) Let BC = *h* metre be the height of the tower and distance from observer to the foot of the tower be AB = 100 m.



In right-angled ΔABC,

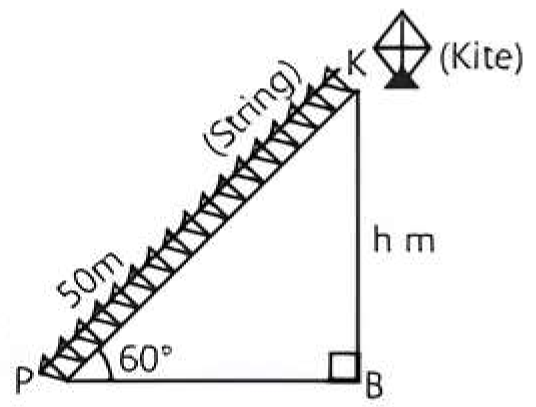
$$\tan 60° = \frac{BC}{AB}$$

$\Rightarrow \quad \sqrt{3} = \frac{h}{100}$

$\Rightarrow \quad h = 100\sqrt{3}$ m

8. (d) Let the string (PK) of a kite (K) in air is PK = 50m long and it makes an angle KPB = 60° with the horizontal PB.

Suppose the height of the Kite from the ground (KB) = h m.



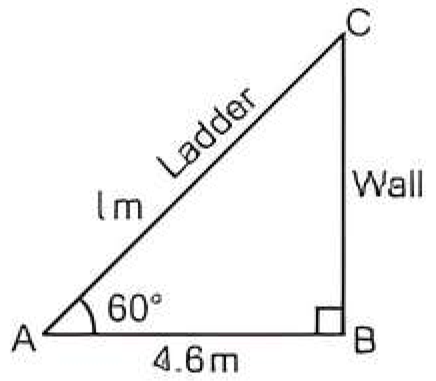
Now, In right-angled Δ PBK,

$$\sin 60^\circ = \frac{KB}{PK}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{h}{50} \Rightarrow h = \frac{50\sqrt{3}}{2} = 25\sqrt{3}\text{m.}$$

So, required height is $25\sqrt{3}$m.

9. (d) Let BC be the height of the wall and AC = l m be the length of the ladder leaning against a wall.



Ladder AC makes an angle of elevation of 60° to the wall *i.e.* ∠CAB = 60°

Let AB = 4.6 m be the foot of the ladder from the wall.

In right-angled ΔABC,

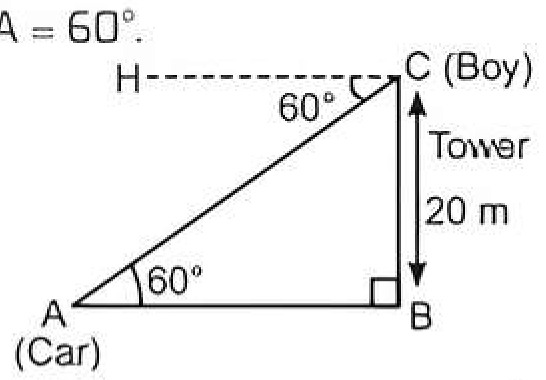
$$\cos 60^\circ = \frac{AB}{AC} \qquad \left(\because \cos\theta = \frac{B}{H}\right)$$

$$\Rightarrow \quad \frac{1}{2} = \frac{4.6}{l}$$

$$\Rightarrow \quad l = 9.2 \text{ m}$$

10. (b) Let BC = 20 m be the height of the tower. Let A be the position of car and C be the position of boy.

At point C, boy makes an angle of depression of 60° *i.e.*, ∠HCA = 60°.



Here, ∠BAC = ∠HCA = 60° (alternate angles)

In right-angled ΔABC,

$$\tan 60^\circ = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

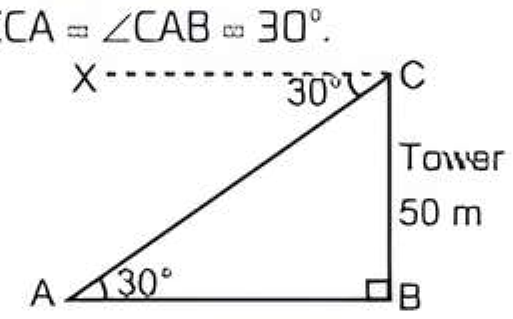
$$\Rightarrow \quad \sqrt{3} = \frac{20}{AB}$$

$$\Rightarrow \quad AB = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{20}{3} \times 1.73$$

$$= 6.67 \times 1.73 = 11.54 \text{ m}$$

11. (c) Let A be the position of an object and BC = 50 m be the height of tower.

Then ∠XCA = ∠CAB = 30°. (alternate angles)



In right-angled ΔABC,

$$\tan 30^\circ = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50}{AB}$$

$$\Rightarrow \quad AB = 50\sqrt{3} \text{ m}$$

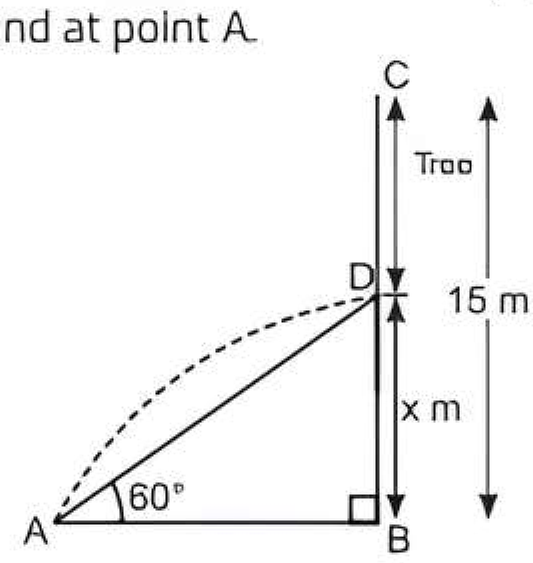
12. (a) In right-angled ΔABC,

$$\tan 30^\circ = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{120}{AB}$$

$$\Rightarrow \quad AB = 120\sqrt{3} \text{ m}$$

13. (a) Let BC = 15 m be the height of the tree. Let at point D, tree breaks and touches the top point C of tree to the ground at point A.



TR!CK

The broken part CD of tree is equal to the slope line AD, i.e., CD = AD.

Let BD = x m be the height of broken tree.

Then CD = AD = 15 − x.

Given, broken part of tree CD makes an angle of 60° with the ground *i.e.* ∠DAB = 60°.

In right-angled ΔABD,

$$\sin 60^\circ = \frac{BD}{AD}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{x}{15 - x}$$

$$\Rightarrow \quad 15\sqrt{3} - \sqrt{3}x = 2x$$

$$\Rightarrow \quad x(2 + \sqrt{3}) = 15\sqrt{3}$$

$$\Rightarrow \quad x = \frac{15\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

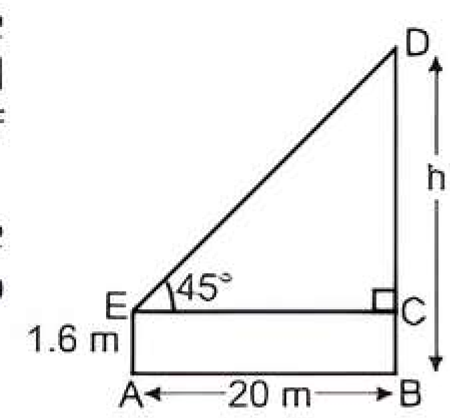
$$= \frac{30\sqrt{3} - 45}{(2)^2 - (\sqrt{3})^2} = \frac{30 \times 1.73 - 45}{4 - 3}$$

$$(\because (a+b)(a-b) = a^2 - b^2)$$

$$= 51.9 - 45 = 6.9 \text{ m}$$

14. (a) Let AE = 1.6 m be the height of an observer and BD = h m be the height of the tower.

Let EC = AB = 20 m be the distance from observer to the tower.



In right-angled ΔECD,

$$\tan 45^\circ = \frac{CD}{EC}$$

$$(\because AB = EC = 20 \text{ m and } AE = BC = 1.6 \text{ m})$$

$$\Rightarrow \quad 1 = \frac{h-1.6}{20} \qquad \left(\because CD = BD - BC = h - 1.6\right)$$

$$\Rightarrow \quad h - 1.6 = 20$$

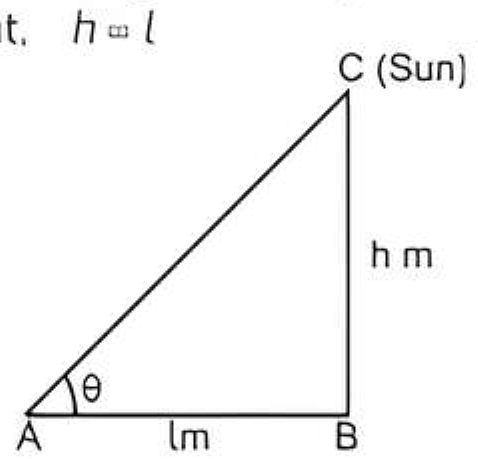
$$\Rightarrow \quad h = 21.6 \text{ m}$$

COMMON ERROR

Sometimes students make an error of taking an angle from point A instead of taking at point E. So, continuous practice is required to make stronger concept.

15. (a) **Assertion (A):** Let BC = h m be the height of the pole and AB = l m be the length of the shadow. Let the Sun makes an angle θ from point A.

Given that, $h = l$



In right-angled triangle ABC,

$$\tan\theta = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \tan\theta = \frac{h}{l} = \frac{l}{l} \qquad [\because h = l \text{ (given)}]$$

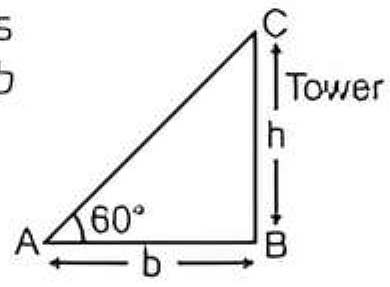
$$\Rightarrow \quad \tan\theta = 1 = \tan 45° \quad \Rightarrow \quad \theta = 45°$$

So, Assertion (A) is true.

Reason (R): It is also true that $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

16. (d) **Assertion (A):** Let BC = h units be the height of tower and AB = b units be the base of the tower.



Then $\tan 60° = \frac{BC}{AB}$

$$\Rightarrow \quad \tan 60° = \frac{h}{b} \qquad \ldots(1)$$

If we tripled the height and base of tower *i.e.*, BC = $3h$ and AB = $3b$, then angle will be

$$\tan\theta = \frac{BC}{AB} = \frac{3h}{3b} \quad \Rightarrow \quad \tan\theta = \frac{h}{b}$$

$$\Rightarrow \quad \tan\theta = \tan 60° \qquad [\text{from eq. (1)}]$$

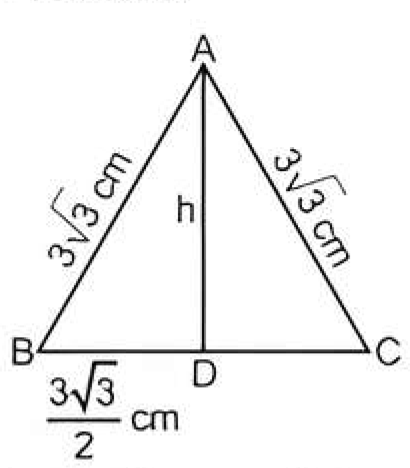
$$\Rightarrow \quad \theta = 60°.$$

which is not tripled the original angle.

So, Assertion (A) is false.

Reason (R): Let ABC be an equilateral triangle. Then

$$AB = BC = CA = 3\sqrt{3} \text{ cm}$$



Let h be the altitude of an equilateral triangle.

TiP

Altitude of an equilateral triangle divides the base into two equal parts.

$$\therefore \quad BD = DC = \frac{3\sqrt{3}}{2} \text{ cm}$$

In right-angled ΔADB, use Pythagoras theorem.

$$AD = \sqrt{(AB)^2 - (BD)^2} = \sqrt{(3\sqrt{3})^2 - \left(\frac{3\sqrt{3}}{2}\right)^2}$$

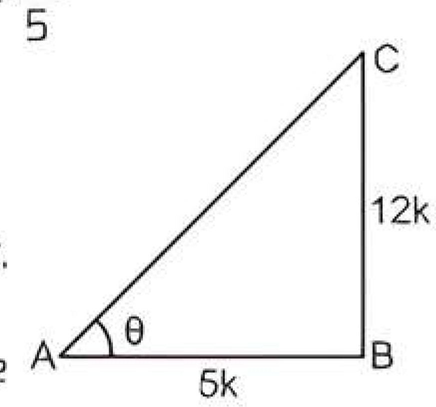
$$= \sqrt{27 - \frac{27}{4}} = \sqrt{\frac{81}{4}} = \frac{9}{2} = 4.5 \text{ cm}$$

So, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

17. (a) **Assertion (A):** Given $\tan\theta = \frac{12}{5}$

$$\Rightarrow \quad \tan\theta = \frac{12}{5} = \frac{BC}{AB}$$



Let BC = $12k$ and AB = $5k$, where k is a constant.

In right-angled ΔABC, use Pythagoras theorem,

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{(5k)^2 + (12k)^2}$$

$$= \sqrt{25k^2 + 144k^2} = \sqrt{169k^2}$$

$$= 13k = 13 \text{ units} \quad (\text{Consider } k = 1)$$

So, Assertion (A) is true.

Reason (R): It is a true relation that

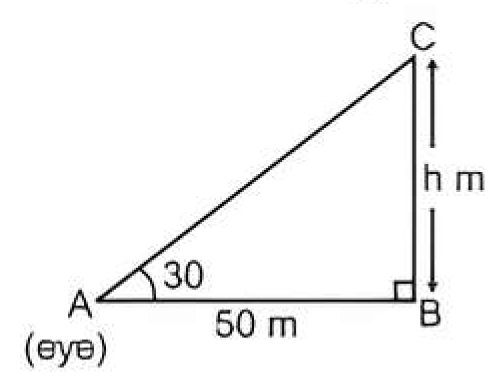
In a right-angled triangle,

$(\text{Hypotenuse})^2 = (\text{Side})^2 + (\text{Base})^2$

So, Reason (R) is true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

18. (a) **Assertion (A):** Let A be the position of observer eye and BC = h m be the height of the tower.



Let AB = 50 m be distance between observer's eye and foot of the tower.

In right-angled ΔABC,

$$\tan 30° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50}$$

$$\Rightarrow \quad h = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{50}{3} \times \sqrt{3} \text{ m}$$

So, Assertion (A) is true.

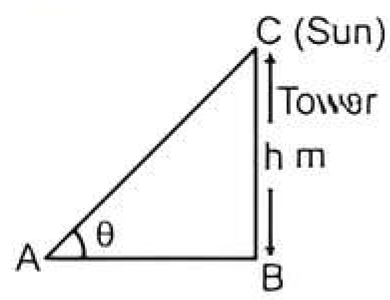
Reason (R): It is true to say that while solving the problem of angle of elevation/depression, triangle should be a right-angled triangle.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

19. line

20. depression

21. Let BC = h m be the height of the tower and C be the position of Sun. The length of the shadow will be AB = $\sqrt{3}h$ m.



Let the elevation of Sun from point A is $\angle CAB = \theta$.

In right-angled ΔABC,

$$\tan\theta = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

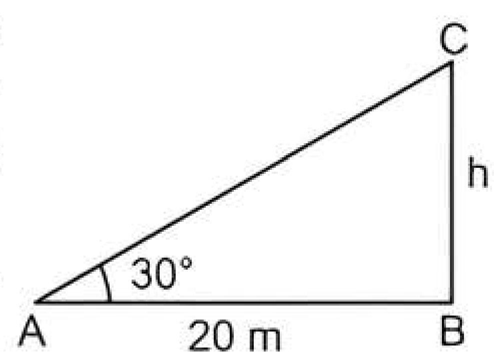
$$= \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$$

$$= \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

22. Let BC = h m be the height of the tower. Let A be the foot of the point such that AB = 20 m.

Also angle of elevation is $\angle BAC = 30^\circ$.



In right-angled triangle,

$$\tan 30^\circ = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20}$$

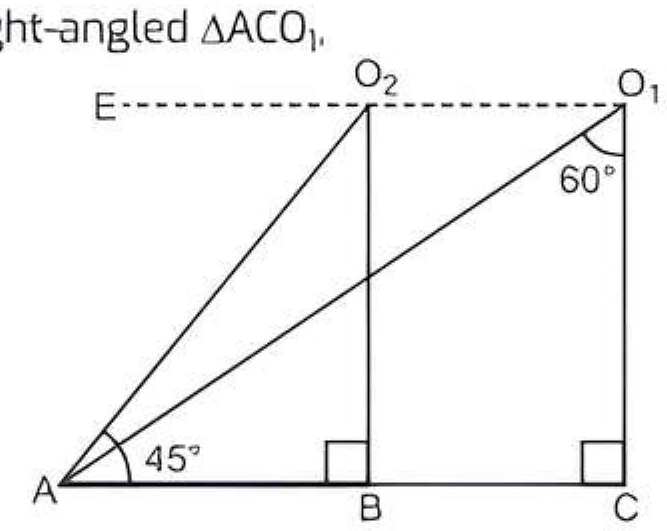
$$\Rightarrow \quad h = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \text{m}$$

$$= \frac{20}{3} \times 1.73 = \frac{34.6}{3} \text{m}$$

$$= 11.53 \text{ m}$$

Hence, height of the tower is 11.53 m.

23. In right-angled ΔACO_1,



$$\angle O_1AC + \angle ACO_1 + \angle CO_1A = 180^\circ$$

(by angle sum property)

$$\Rightarrow \quad \angle O_1AC + 90^\circ + 60^\circ = 180^\circ$$

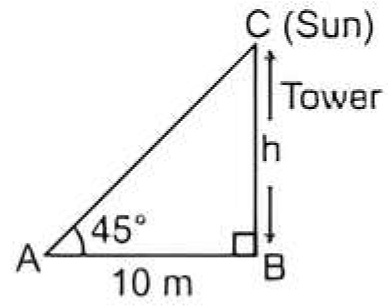
$$\Rightarrow \quad \angle O_1AC = 180^\circ - 150^\circ = 30^\circ$$

Here, $\angle EO_1A = \angle O_1AC = 30^\circ$ (alternate angles)

Also, $\angle EO_2A = \angle O_2AB = 45^\circ$ (alternate angles)

Hence, angles of depressions from points O_1 and O_2 are respectively 30° and 45°.

24. Let C be the position of Sun and BC = h m be the height of the tower. Let AB = 10 m be the length of the Sun.



In right-angled ΔABC,

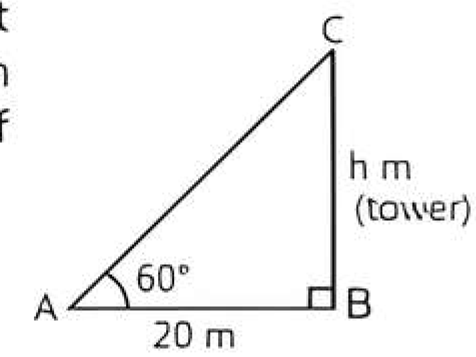
$$\tan 45^\circ = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad 1 = \frac{h}{10}$$

$$\Rightarrow \quad h = 10 \text{ m}$$

Hence, given statement is true.

25. Let BC = h metre be the height of the tower and AB = 20 m be the length of shadow of tower.



In right-angled ΔABC,

$$\tan 60^\circ = \frac{BC}{AB}$$

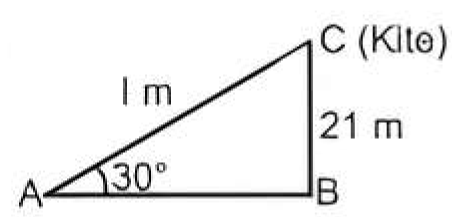
$$\Rightarrow \quad \sqrt{3} = \frac{h}{20}$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ m}$$

Hence, given statement is false.

26. Let C be the position of the kite and A be the position of the boy.

Let AC = l m be the length of the string and BC = 21m be the height of the kite. Then string of kite makes an angle $\angle CAB = 30^\circ$.



In right-angled ΔABC,

$$\sin 30^\circ = \frac{BC}{AC} = \frac{21}{l} \qquad \left(\because \sin\theta = \frac{P}{H}\right)$$

$$\Rightarrow \quad \frac{1}{2} = \frac{21}{l}$$

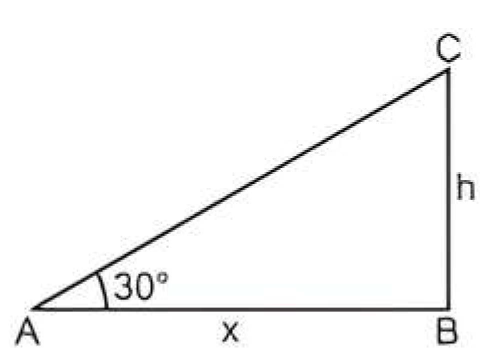
$$\Rightarrow \quad l = 21 \times 2 = 42 \text{ m}$$

Hence, given statement is false.

27. Let initially height of tower be h m and AB = x m.

In right angled ΔABC,

$$\tan 30^\circ = \frac{h}{x}$$



If the height of tower is doubled *i.e.*, BC = $2h$, then

$$\tan\theta = \frac{2h}{x}$$

Here we see that angle is not doubled when height is doubled.

Hence, given statement is false.

Case Study Based Questions

Case Study 1

There is fire incident in the house. The house door is locked so, the fireman is trying to enter the house from the window. He places the ladder against the wall such that its top reaches the window as shown in the figure.

Based on the above information, solve the following questions:

Q 1. If window is 6 m above the ground and angle made by the foot of ladder to the ground is 30°, then length of the ladder is:

a. 8 m b. 10 m c. 12 m d. 14 m

Q 2. If fireman place the ladder 5 m away from the wall and angle of elevation is observed to be 30°, then length of the ladder is:

a. 5 m b. $\frac{10}{\sqrt{3}}$ m c. $\frac{15}{\sqrt{2}}$ m d. 20 m

Q 3. If fireman places the ladder 2.5 m away from the wall and angle of elevation is observed to be 60°, then find the height of the window: (Take $\sqrt{3} = 1.73$)

a. 4.325 m b. 5.5 m c. 6.3 m d. 2.5 m

Q 4. If the height of the window is 8 m above the ground and angle of elevation is observed to be 45°, then horizontal distance between the foot of ladder and wall is:

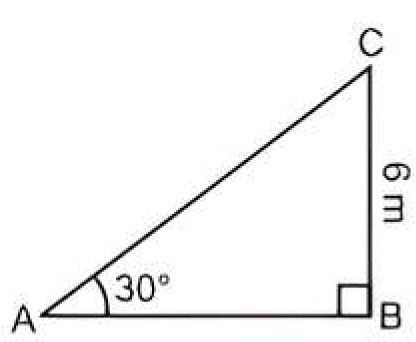
a. 2 m b. 4 m c. 6 m d. 8 m

Q 5. If the fireman gets a 9 m long ladder and window is at 6 m height, then how far should the ladder be placed?

a. 5 m b. $3\sqrt{5}$ m c. 3 m d. 4 m

Solutions

1. Let AC be the length of the ladder



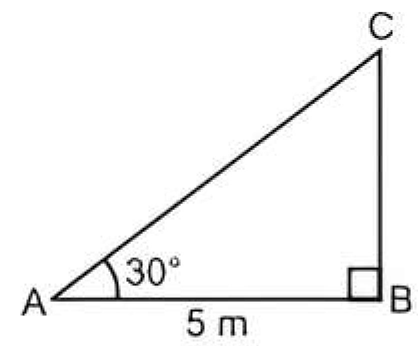
In right-angled ΔABC,

$$\sin 30° = \frac{BC}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{6}{AC} \Rightarrow AC = 12 \text{ m}$$

So, option (c) is correct.

2. In right-angled ΔABC, $\cos 30° = \frac{AB}{AC}$



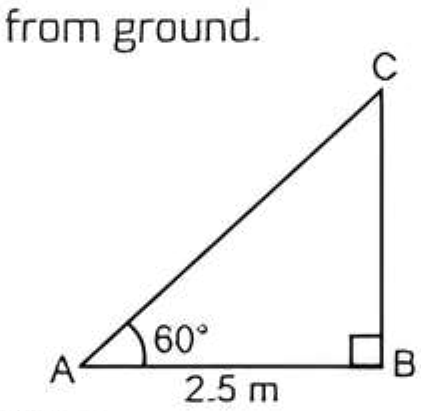
$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{5}{AC}$$

$$\Rightarrow AC = \frac{10}{\sqrt{3}} \text{ m}$$

So, option (b) is correct.

3. Let BC be the height of window from ground.

In right-angled ΔABC,



$$\tan 60° = \frac{BC}{AB}$$

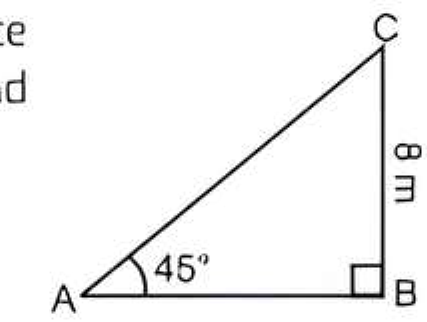
$$\Rightarrow \sqrt{3} = \frac{BC}{2.5}$$

$$\Rightarrow BC = 2.5 \times 1.73 = 4.325 \text{ m}$$

So, option (a) is correct.

4. Let AB be the horizontal distance between the foot of ladder and wall.

In right-angled ΔABC,



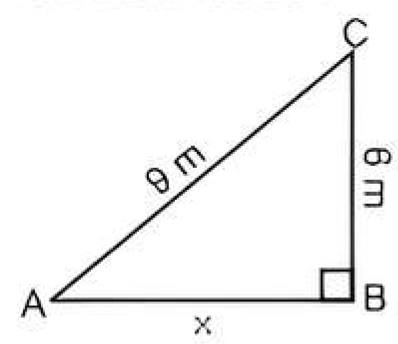
$$\tan 45° = \frac{BC}{AB}$$

$$\Rightarrow 1 = \frac{8}{AB}$$

$$\Rightarrow AB = 8 \text{ m}$$

So, option (d) is correct.

5. Let the required distance be x.



In right-angled ΔABC,

$$(9)^2 = x^2 + (6)^2$$

(By Phthagoras theorem)

$$\Rightarrow 81 - 36 = x^2$$

$$\Rightarrow 45 = x^2$$

$$\Rightarrow x = 3\sqrt{5} \text{ m}$$

So, option (b) is correct.

Case Study 2

A group of students of class–X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name is Delhi Memorial, originally called All-India War Memorial, Monumental Sandstone Arch in New Delhi is dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

Based on the given information, solve the following questions:

Q 1. What is the angle of elevation, if they are standing at a distance of $42\sqrt{3}$ m away from the monument?

a. 0° b. 30° c. 45° d. 60°

Q 2. They want to see the tower (monument) at an angle of 60°. So, they want to know the distance where they should stand and hence find the distance. [Use $\sqrt{3} = 1.732$]

a. 24.24 m b. 20.12 m c. 42 m d. 25.64 m

Q 3. If the altitude of the Sun is at 30°, then the height of the vertical tower that will cast a shadow of length 30 m is:

a. $10\sqrt{3}$ m b. $\frac{10}{\sqrt{3}}$ m c. $\frac{20}{\sqrt{3}}$ m d. $20\sqrt{3}$ m

Q 4. The ratio of the length of a rod and its shadow is $24 : 8\sqrt{3}$. The angle of elevation of the Sun is:

a. 30° b. 60°
c. 45° d. 90°

Q 5. The angle formed by the line of sight with the horizontal when the object viewed is above the horizontal level, is:

a. angle of elevation b. angle of depression
c. corresponding angle d. complete angle

Solutions

1. Let the angle of elevation be θ.

Given that,

Height of the monument (BC) = 42 m

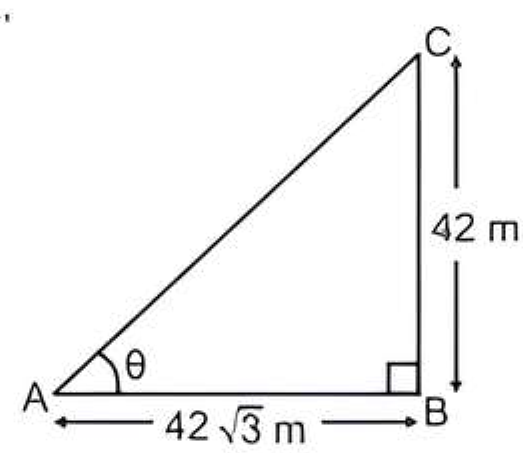
and $AB = 42\sqrt{3}$ m

Now, in right-angled ΔABC,

$$\tan\theta = \frac{BC}{AB} = \frac{42}{42\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\therefore \theta = 30^\circ$$



So, option (b) is correct.

COMMON ERR(!)R

Students take the value of $\frac{1}{\sqrt{3}} = \tan 60^\circ$ *in precocity.*

But it is wrong. The correct value of $\frac{1}{\sqrt{3}} = \tan 30^\circ$.

2. Let the required distance be x m.

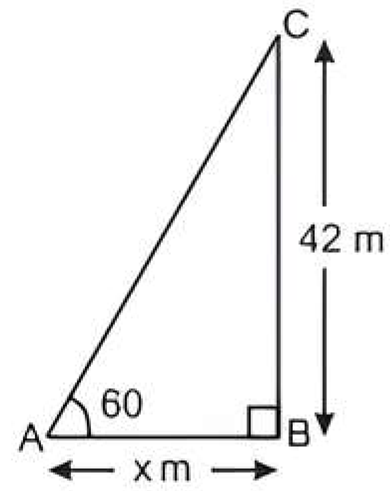
Given, angle of elevation (θ) = 60°

and height of the monument (BC) = 42 m

TiP

Memorize the values of trigonometric angles properly and do practice more.

Now, in right-angled ΔABC,



$$\tan\theta = \frac{BC}{AB}$$

$$\Rightarrow \tan 60^\circ = \frac{42}{x}$$

$$\Rightarrow \sqrt{3} = \frac{42}{x}$$

$$\Rightarrow x = \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{42\sqrt{3}}{3} = 14\sqrt{3}$$

$$= 14 \times 1.732 = 24.24 \text{ m}$$

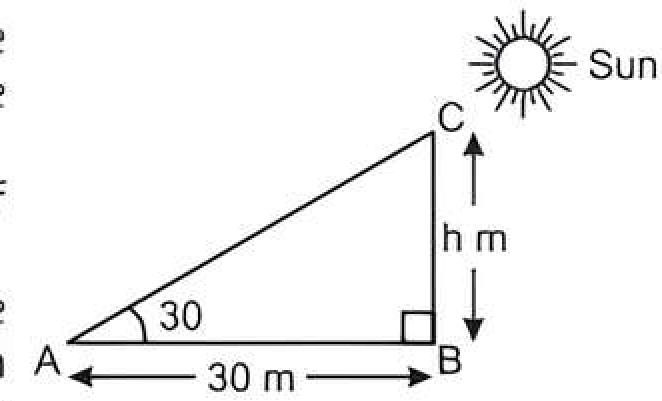
So, option (a) is correct.

3. Let the height of the vertical tower be h m.

Given angle of elevation (θ) = 30°

and length of the shadow (AB) = 30 m

Now, In right-angled ΔABC.



$$\tan\theta = \frac{BC}{AB}$$

$$\Rightarrow \tan 30^\circ = \frac{h}{30}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ m}$$

So, option (a) is correct.

COMMON ERR(!)R

Some students confused the values of tan 30° and tan 60°. They take wrong value in haste.

4. Given, the ratio of the length of a rod and its shadow is $24 : 8\sqrt{3}$.

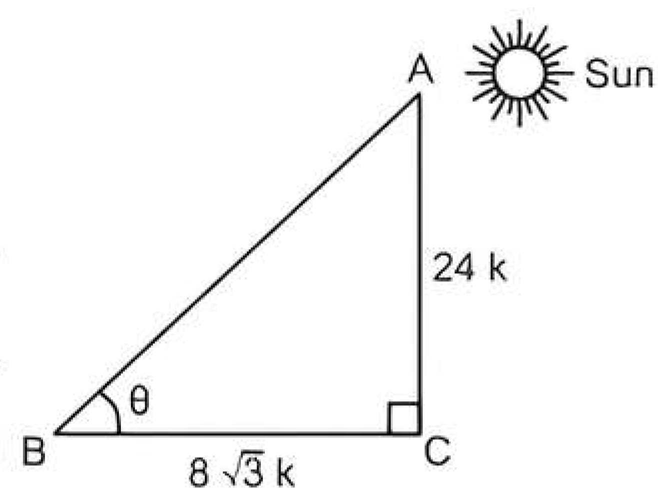
Let $AC = 24k$

and $BC = 8\sqrt{3}k$

where k is a positive integer.

Now, let angle of elevation of the Sun is θ.

In right-angled ΔACB;



$$\tan\theta = \frac{AC}{BC}$$

$$= \frac{24k}{8\sqrt{3}k} = \frac{3}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \tan 60^\circ$$

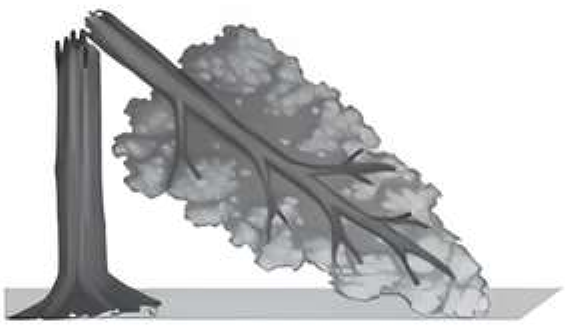
$$\therefore \quad \theta = 60^\circ$$

So, option (b) is correct.

5. The angle formed by the line of sight with the horizontal, when the object viewed is above the horizontal level, is angle of elevation.
So, option (a) is correct.

Case Study 3

Suppose a straight vertical tree is broken at some point due to storm and the broken part is inclined at a certain distance from the foot of the tree.



Based on the above information, solve the following questions:

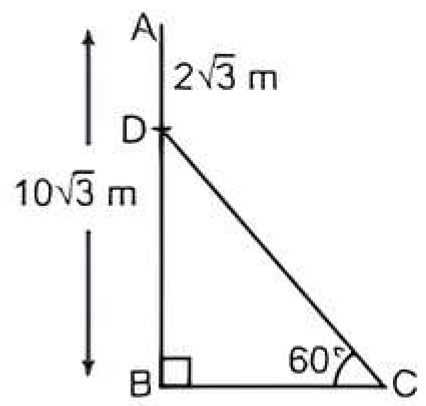
Q 1. If the top of upper part of broken tree touches ground at a distance of 30 m (from the foot of the tree) and makes an angle of inclination 30°, then find the height of remaining part of the tree.

Q 2. Find the height of the straight vertical tree.

Q 3. If the height of a tree is 6 m, which is broken by wind in such a way that its top touches the ground and makes an angle 30° with the ground. Find the length of broken part of the tree.

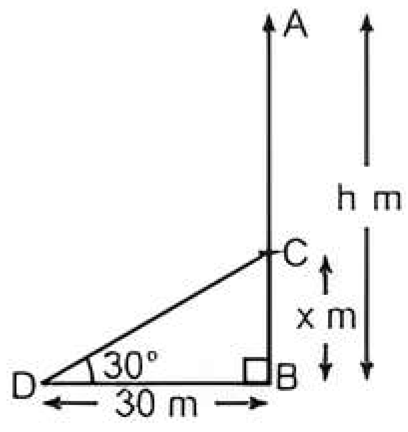
OR

If $AB = 10\sqrt{3}$ m and $AD = 2\sqrt{3}$ m, then find CD.



Solutions

1. Let AB be the tree of height h m and let it broken at height of x m, as shown in figure.
Clearly $CD = AC = (h - x)$ m
Now, in right-angled ΔCBD, we have



$$\tan 30^\circ = \frac{BC}{BD} = \frac{x}{30}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{30}$$

$$\Rightarrow \quad x = \frac{30}{\sqrt{3}}$$

$$= \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\text{ m}$$

Thus, the height of remaining part of the tree is $10\sqrt{3}$ m.

2. In right-angled ΔCBD,

$$\cos 30^\circ = \frac{DB}{DC} = \frac{30}{DC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{30}{DC}$$

$$\Rightarrow \quad DC = \frac{60}{\sqrt{3}} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3}$$

$$\Rightarrow \quad DC = 20\sqrt{3}\text{ m}$$

from part (1), $BC = x = 10\sqrt{3}$ m

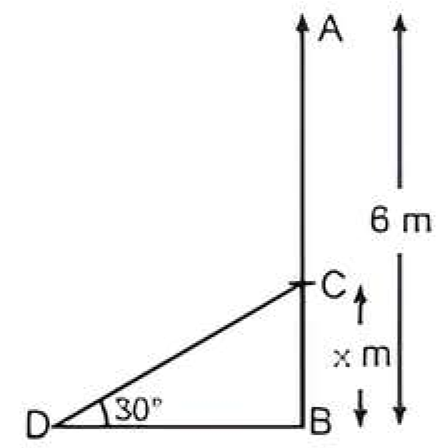
Thus, the height of the straight vertical tree

$$AB = DC + BC$$
$$= 20\sqrt{3} + 10\sqrt{3} = 30\sqrt{3}\text{ m}$$

3. Here, $h = 6$ m and $\theta = 30^\circ$

$$\therefore \quad DC = AC = (6 - x)\text{ m}$$

Now, in right-angled ΔBCD, we have



$$\sin 30^\circ = \frac{BC}{CD}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{x}{6 - x}$$

$$\Rightarrow \quad 6 - x = 2x \Rightarrow 3x = 6 \Rightarrow x = 2$$

So, broken part of tree, $AC = 6 - x = 6 - 2 = 4$ m.

Or

Clearly, $BD = AB - AD$

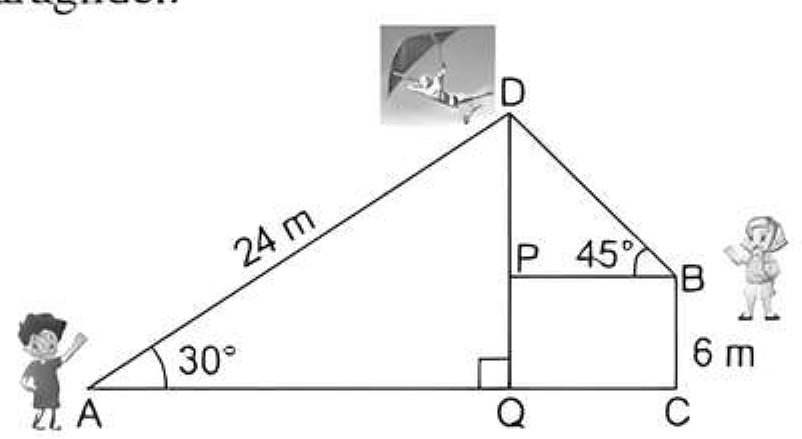
$$= (10\sqrt{3} - 2\sqrt{3})\text{ m} = 8\sqrt{3}\text{ m}$$

Now, in right-angled ΔBCD, we have

$$\sin 60^\circ = \frac{BD}{DC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8\sqrt{3}}{DC} \Rightarrow DC = 16\text{ m}$$

Case Study 4

Sandeep and his sister Dolly visited at their uncle's place–Birth, Himachal Pradesh. During day time Sandeep, who is standing on the ground spots a paraglider at a distance of 24 m from him at an elevation of 30°. His sister Dolly is also standing on the roof of a 6 m high building, observes the elevation of the same paraglider as 45°. Sandeep and Dolly are on the opposite sides of the paraglider.



Based on the given information, solve the following questions:

Q 1. Find the distance of paraglider from the ground.

Q 2. Find the value of PD.

Q 3. Find the distance between the paraglider and the Dolly.

Or

Find the distance between Sandeep and base of the building.

Solutions

1. In the right-angled ΔAQD, we have

$\sin 30° = \frac{DQ}{AD} \Rightarrow \frac{1}{2} = \frac{DQ}{24}$

$\Rightarrow \quad DQ = 12 \text{ m}$

Thus, distance of paraglider from the ground is 12 m.

2. We have $PQ = BC = 6$ m

Now, as $DQ = 12$ m

$\therefore \quad DP = DQ - PQ = 12 - 6 = 6$ m

3. In right-angled ΔBPD, we have

$$\sin 45° = \frac{DP}{BD} \Rightarrow \frac{1}{\sqrt{2}} = \frac{6}{BD} \Rightarrow \quad BD = 6\sqrt{2} \text{ m}$$

Thus, the distance of paraglider from the girl is $6\sqrt{2}$ m.

Or

In right-angled ΔAQD, we have

$$\cos 30° = \frac{AQ}{AD}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AQ}{24} \Rightarrow AQ = 12\sqrt{3} \text{ m}$$

In right-angled ΔBPD, we have

$$\cos 45° = \frac{BP}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{BP}{6\sqrt{2}} \Rightarrow BP = 6\text{m}$$

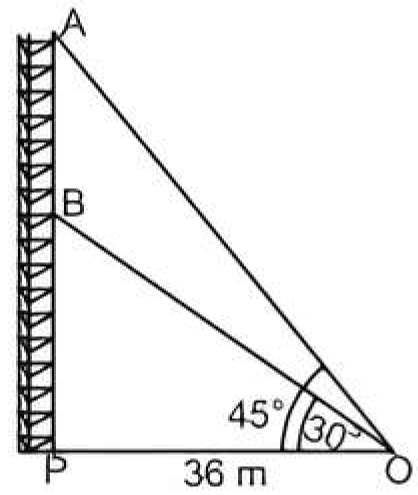
Thus, the distance between Sandeep and base of the building $= AQ + BP$

$$= 12\sqrt{3} + 6 = 6\,(2\sqrt{3} + 1) \text{ m}.$$

Case Study 5

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is $30°$ and the angle of elevation of the top of Section A is $45°$.



Based on the given information, solve the following questions: [CBSE 2023]

Q 1. Find the length of the wire from the point O to the top of Section B.

Q 2. Find the height of the Section A from the base of the tower.

Q 3. Find the distance AB.

Or

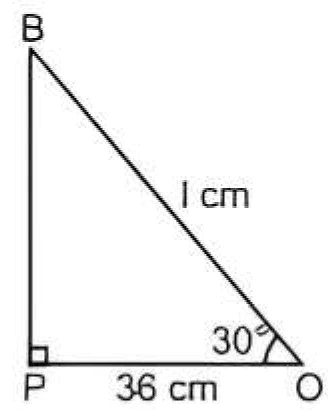
Find the area of ΔOPB.

Solutions

1. Let the length of the wire from the point O to the top of section B, *i.e.*, $OB = l$ *m*.

Given, $OP = 36$ cm and $\angle BOP = 30°$

Now in right-angled ΔBPO,



$$\cos 30° = \frac{OP}{OB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{36}{l}$$

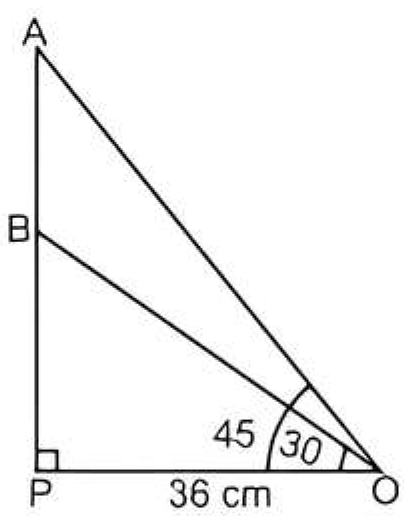
$$\Rightarrow l = \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{72\sqrt{3}}{3} = 24\sqrt{3}$$

So, required length is $24\sqrt{3}$ cm.

2. Given, $\angle AOP = 45°$ and

$OP = 36$ cm

Now in right-angled ΔAPO,



$$\tan 45° = \frac{AP}{OP}$$

$$\Rightarrow \quad 1 = \frac{AP}{36}$$

$$\Rightarrow \quad AP = 36 \text{ cm}$$

$\therefore$ The height of the Section A from the base of the tower $= AP = 36$ cm.

3. Now, in right-angled ΔBPO,

$$\tan 30° = \frac{BP}{OP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BP}{36}$$

$$\Rightarrow \quad BP = \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{36\sqrt{3}}{3} = 12\sqrt{3} \text{ cm}$$

$$AB = AP - BP \quad [\because AP = 36 \text{ cm}]$$

$$= 36 - 12\sqrt{3} = 12(3 - \sqrt{3}) \text{ cm}$$

So, required distance AB is $12\,(3 - \sqrt{3})$ cm.

Or

Since, ΔBPO is a right-angled triangle.

$\therefore$ Area of $\Delta OPB = \frac{1}{2} \times \text{base} \times \text{height}$

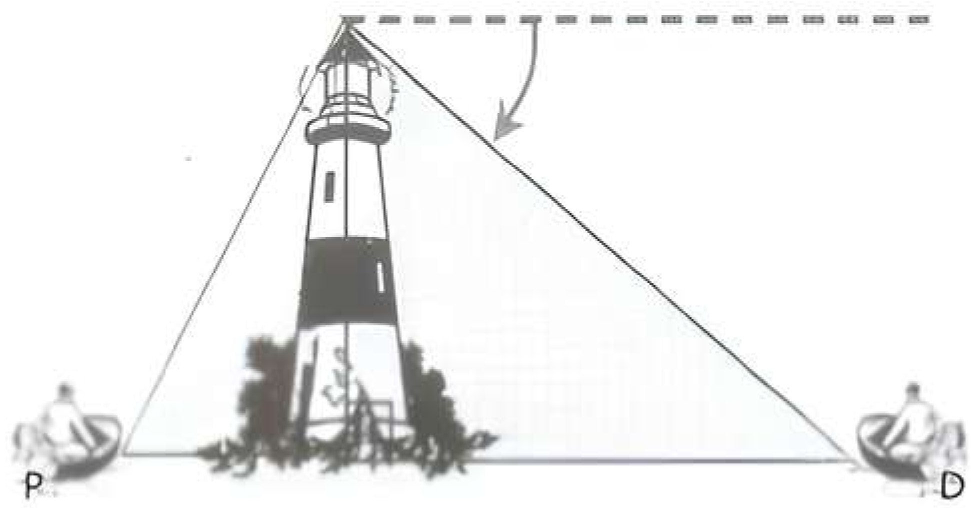
$= \frac{1}{2} \times OP \times BP = \frac{1}{2} \times 36 \times 12\sqrt{3}$

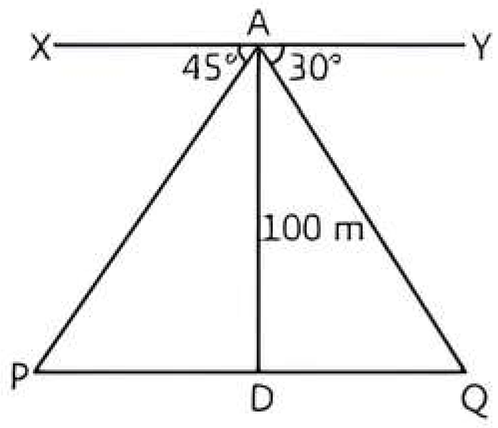
$= 216\sqrt{3}\ \text{cm}^2.$

Case Study 6

A boy is standing on the top of light house. He observed that boat P and boat Q are approaching the light house from opposite directions. He finds that angle of depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of the light house is 100 m.

[CBSE SQP 2023-24]





Based on the above information, answer the following questions.

Q 1. What is the measure of $\angle APD$?

Q 2. If $\angle YAQ = 30°$, then $\angle AQD$ is also 30°, why?

Q 3. Find length of PD.

Or

Find length of QD.

Solutions

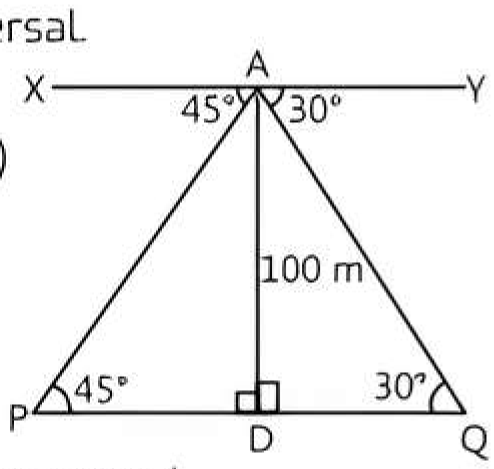
1. Let a boy is standing on the top (A) of light house (AD).

Here XY|| PQ and AP is traversal.

$\therefore \quad \angle APD = \angle PAX$

(Alternative interior angles)

$\Rightarrow \quad \angle APD = 45°.$



2. Given, $\angle YAQ = 30°$

$\Rightarrow \quad \angle AQD = 30°$

Because, XY||PQ and AQ is a traversal.

So, alternate interior angles are equal.

$\therefore \quad \angle YAQ = \angle AQD.$

3. In right-angled ΔADP,

$$\tan 45° = \frac{AD}{PD}$$

$$\Rightarrow \quad 1 = \frac{100}{PD}$$

$$\Rightarrow \quad PD = 100\ \text{m}.$$

$\therefore$ Boat P is 100 m from the light house.

Or

In right-angled ΔADQ,

$$\tan 30° = \frac{AD}{DQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{DQ}$$

$$\Rightarrow \quad QD = 100\sqrt{3}\ \text{m}$$

$\therefore$ Boat Q is $100\sqrt{3}$ m from the light house.

Very Short Answer Type Questions

Q 1. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the Sun. *[CBSE 2023]*

Q 2. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$. What is the angle of elevation of the Sun? *[CBSE 2017]*

Q 3. What is the angle of depression of the object at E from the observation point A, if AD = ED? *[CBSE 2017]*

Q 4. The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower, is 30°. Find the height of the tower. *[CBSE 2023]*

Q 5. A building casts a shadow of length $5\sqrt{3}$ m on the ground, when the Sun's elevation is 60°. Find the height of the building.

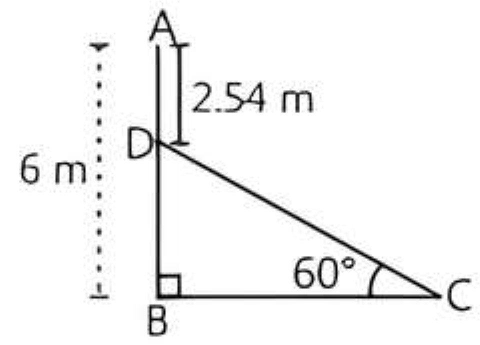
Q 6. A kite is flying, attached to a thread which is 140 m long. The thread makes an angle of 30° with the ground. Find the height of the kite from the ground, assuming that there is no slack in the thread. *[V. Imp.]*

Short Answer Type-I Questions

Q 1. The angle of depression of car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower.

Q 2. Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation θ of the Sun is such that $\tan\theta = \frac{6}{7}$. *[CBSE 2023]*

Q 3. In figure, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If AD = 2.54 m, find the length of the ladder. [Use $\sqrt{3} = 1.73$]. *[CBSE 2016]*



Q 4. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree. *[NCERT EXERCISE; Imp.]*

Q 5. The top of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x:y$. *[CBSE 2015]*

Short Answer Type-II Questions

Q 1. A boy 1.7 m tall is standing on a horizontal ground, 50 m away from a building. The angle of elevation of the top of the building from his eye is 60°. Calculate the height of the building.
[Take $\sqrt{3} = 1.73$] *[CBSE SQP 2022 Term -II]*

Q 2. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the Sun is 60°. Find the angle of elevation of the Sun at the time of the longer shadow. *[CBSE 2017]*

Q 3. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower. *[CBSE 2023]*

Q 4. A person walking 45 m towards a tower in a horizontal line through its base observes that angle of elevation of the top of the tower changes from 45° to 60°. Find the height of the tower. [Use $\sqrt{3} = 1.732$] *[CBSE 2017]*

Q 5. A TV tower stands vertically on the bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from the point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower. *[CBSE 2023]*

Q 6. The angle of elevation of the top of a building from a point A on the ground is 30°. On moving a distance of 30 m towards its base to the point B, the angle of elevation changes to 45°. Find the height of the building and the distance of its base from point A. (Use $\sqrt{3} = 1.732$) *[CBSE 2023]*

Q 7. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, then find the width of the river. *[CBSE 2022 Term-II]*

Q 8. Two vertical poles of different heights are standing 20m away from each other on the level ground. The angle of elevation of the top of the first pole from the foot of the second pole is 60° and angle of elevation of the top of the second pole from the foot of the first pole is 30°. Find the difference between the heights of two poles. [Take $\sqrt{3} = 1.73$] *[CBSE SQP 2022 Term-II]*

Q 9. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 min. Find the speed of the boat (in m/min). *[CBSE 2019, 17]*

Long Answer Type Questions

Q 1. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the two cars. [Use $\sqrt{3} = 1.73$] *[CBSE 2023]*

Q 2. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. *[NCERT EXERCISE; CBSE 2020]*

Q 3. An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. [Use $\sqrt{3} = 1.73$] *[CBSE 2023]*

Q 4. Two ships are approaching a light house from opposite directions. The angles of depression of the two ships from the top of a lighthouse are 30° and 45°. If the distance between the two ships is 100 m, find the height of the lighthouse. [Use $\sqrt{3} = 1.732$]

Q 5. A straight highway leads to the foot of a tower. A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60°, which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. [Use $\sqrt{3} = 1.73$] *[CBSE 2023]*

Q 6. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$] *[NCERT EXERCISE; CBSE 2018, 17]*

Q 7. From the top of a building 60 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower. Also, find the distance between the building and the tower. [Use $\sqrt{3} = 1.732$] *[CBSE 2023]*

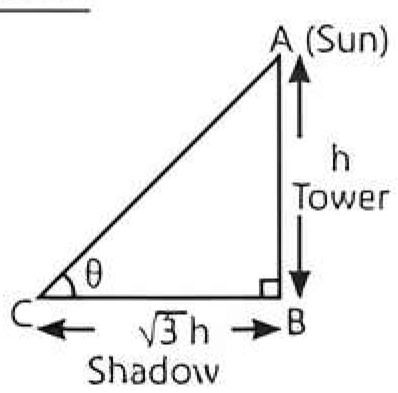
Solutions

Very Short Answer Type Questions

1. Let AB be the tower and BC be its shadow.

Let angle of elevation of the Sun be θ.

In right-angled Δ ABC.



$$\tan\theta = \frac{AB}{BC} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \tan\theta = \frac{h}{h\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

Hence, the elevation of the Sun is 30°.

COMMON ERROR

Students take the value of $\frac{1}{\sqrt{3}} = \tan 60^\circ$ in haste. But it is wrong. So, the correct value of $\frac{1}{\sqrt{3}} = \tan 30^\circ$.

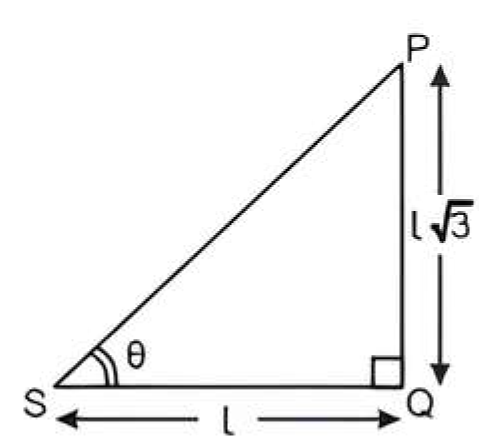
2. Given that, the ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$.

Let, tower height $(PQ) = l\sqrt{3}$

and its shadow length $(SQ) = l$

Let $\angle PSQ = \theta$ (angle of elevation of the Sun)

Now, in right-angled ΔPQS,



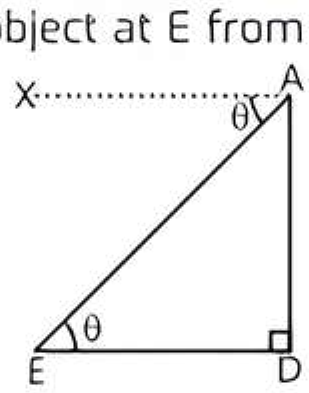
$$\tan\theta = \frac{PQ}{SQ} = \frac{l\sqrt{3}}{l} = \sqrt{3}$$

$\Rightarrow \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$

So, the angle of elevation of the Sun is 60°.

3. Let the angle of depression of the object at E from the observation point A is θ.

In right-angled ΔADE.



$$\tan\theta = \frac{AD}{ED} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$= \frac{AD}{AD} = 1 \qquad (\because AD = ED)$$

$$= \tan 45^\circ$$

$$\Rightarrow \quad \theta = 45^\circ$$

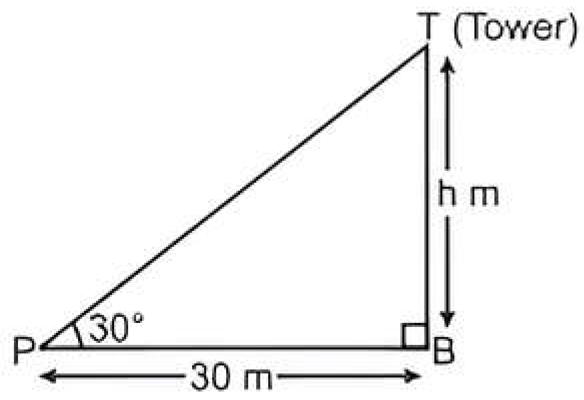
Hence, the angle of depression is 45°.

4. Given, the angle of elevation of the top (T) of a tower TB from a point (P) on the ground which is 30 m away from the foot (B) of the tower, is $\angle BPT = 30^\circ$.

i.e., BP = 30 m and let BT = *h* m

Now, in right-angled ΔTBP,

$$\tan 30^\circ = \frac{BT}{BP} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$



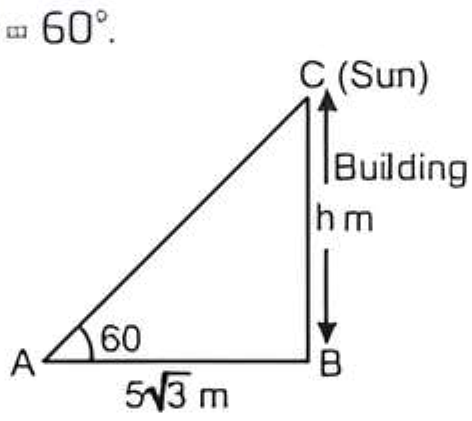
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3}$$

$$\Rightarrow \quad h = 10\sqrt{3} \text{ m}$$

So, height of the tower is $10\sqrt{3}$ m.

5. Let BC = *h* m be the height of the building. $AB = 5\sqrt{3}$ m and $\angle CAB = 60^\circ$.



In right-angled ΔABC,

$$\tan 60^\circ = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

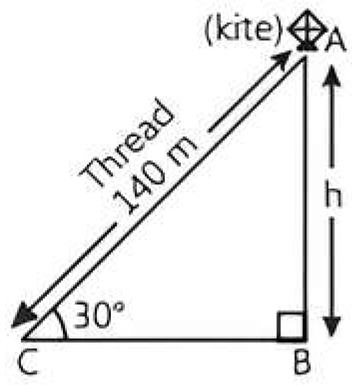
$$\Rightarrow \quad \sqrt{3} = \frac{h}{5\sqrt{3}} \Rightarrow h = 15 \text{ m}$$

Hence, height of the building is 15 m.

6. Let AC be the length of thread and AB be the vertical height of the kite.

TiP

Students should do practice, use of trigonometrical ratios in the triangle.



Let AB = h m, AC = 140 m and $\angle ACB = 30°$.

In right-angled Δ ABC,

$$\sin 30° = \frac{AB}{AC} \qquad \left(\because \sin\theta = \frac{P}{H}\right)$$

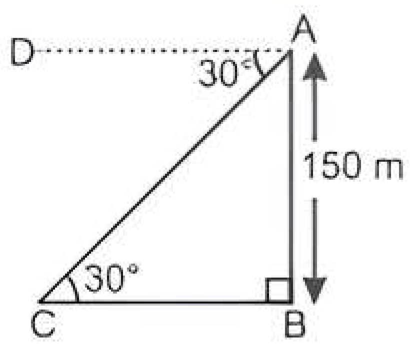
$$\Rightarrow \quad \frac{1}{2} = \frac{h}{140}$$

$$\Rightarrow \quad h = \frac{140}{2} = 70 \text{ m}$$

Hence, the height of the kite is 70 m.

Short Answer Type-I Questions

1. Let AB = 150 m be the height of the tower and angle of depression is $\angle DAC = 30°$.



Then, $\angle ACB = \angle DAC = 30°$ (alternate angles)

In right-angled Δ ABC,

$$\tan 30° = \frac{AB}{BC} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{150}{BC}$$

$$\Rightarrow \quad BC = 150\sqrt{3} \text{ m}$$

Hence, the distance of car from the tower is $150\sqrt{3}$ m.

COMMON ERROR

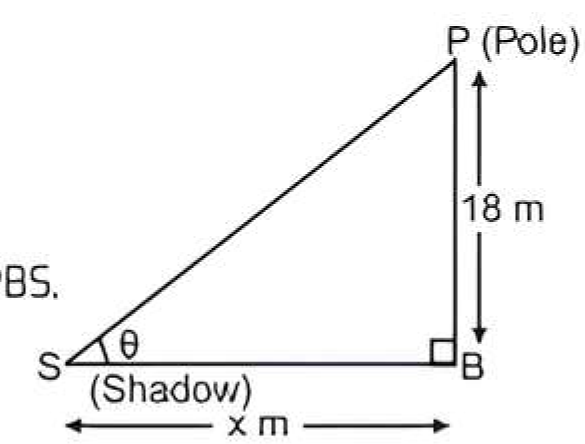
Some candidates are unable to draw the diagram as per the given data and lose their marks.

2. Let the length of the shadow BS = x m on the ground of a pole BP of height 18 m.

Given, BP = 18 m

and $\tan\theta = \frac{6}{7}$

Now in right-angled ΔPBS,



$$\tan\theta = \frac{BP}{BS}$$

$$\Rightarrow \quad \frac{6}{7} = \frac{18}{x}$$

$$\Rightarrow \quad x = \frac{18 \times 7}{6} = 3 \times 7 = 21 \text{m}$$

So, the required length of shadow is 21 m.

3. From figure, BD = AB – AD

$$= 6 - 2.54 = 3.46 \text{ m}$$

In right-angled ΔDBC,

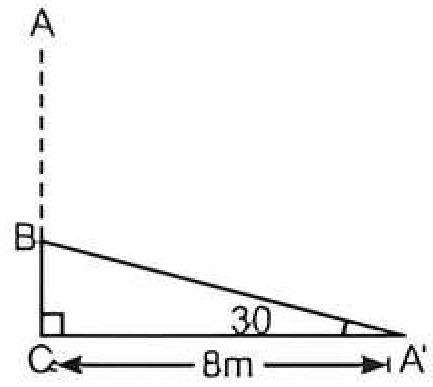
$$\sin 60° = \frac{BD}{CD} = \frac{3.46}{CD}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3.46}{CD}$$

$$\Rightarrow \quad CD = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46 \times 2}{1.73} = 4 \text{ m}$$

Hence, length of the ladder is 4 m.

4. Let AC was the original tree. Due to storm, it was broken into two parts. The broken part A'B is making an angle of 30° with the ground.



In right-angled Δ A'CB,

$$\tan 30° = \frac{BC}{A'C}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$\Rightarrow \quad BC = \frac{8}{\sqrt{3}} \text{m}$$

and $$\cos 30° = \frac{A'C}{A'B}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{8}{A'B}$$

$$\Rightarrow \quad A'B = \frac{16}{\sqrt{3}} \text{m}$$

$\therefore$ Height of the tree = AB + BC = A'B + BC ($\because$ A'B = AB)

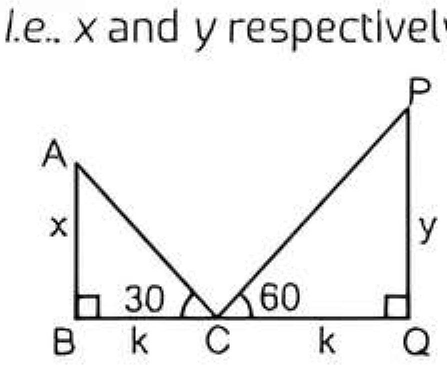
$$= \left[\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right] \text{m}$$

$$= \frac{24}{\sqrt{3}} \text{m} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{m}$$

$$= \frac{24\sqrt{3}}{3} \text{m} = 8\sqrt{3} \text{ m}$$

Hence, the height of the tree is $8\sqrt{3}$m.

5. The base is same for both towers and their heights are given, *i.e.*, x and y respectively.



Let the base of towers be BC = CQ = k.

In right-angled Δ ABC,

$$\tan 30^\circ = \frac{AB}{BC} = \frac{x}{k}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{k}$$

$$\Rightarrow \quad x = \frac{k}{\sqrt{3}} \quad \text{...(1)}$$

In right-angled ΔPQC,

$$\tan 60^\circ = \frac{PQ}{CQ} = \frac{y}{k} \quad \text{...(2)}$$

$$\Rightarrow \quad \sqrt{3} = \frac{y}{k}$$

$$\Rightarrow \quad y = k\sqrt{3}$$

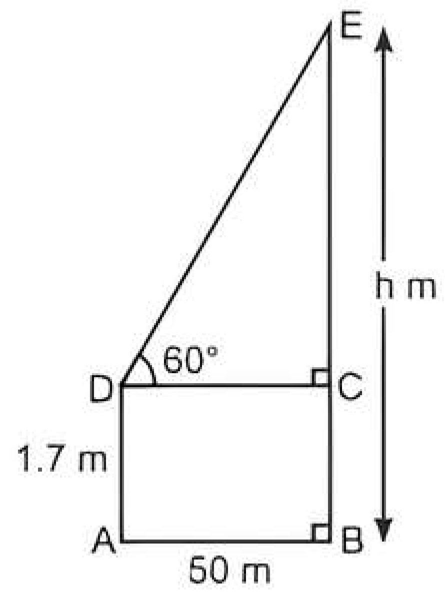
From eqs. (1) and (2), we get

$$\frac{x}{y} = \frac{k}{\sqrt{3}} \times \frac{1}{k\sqrt{3}} = \frac{1}{3}$$

$$\therefore \quad x : y = 1 : 3$$

Short Answer Type-II Questions

1. Let height of the tower be $BE = h$ m and $AD = 1.7$ m be the height of the boy.



Also, given the angle of elevation of the boy to the top of the tower is $\angle CDE = 60^\circ$.

Here, $EC = EB - BC$

$= h - 1.7 \quad (\because BC = AD = 1.7\text{ m})$

Also, $DC = AB = 50$ m.

In right-angled triangle DCE,

$$\tan 60^\circ = \frac{CE}{DC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h - 1.7}{50}$$

$$\Rightarrow \quad 50\sqrt{3} = h - 1.7$$

$$\Rightarrow \quad h = 50 \times 1.73 + 1.7$$

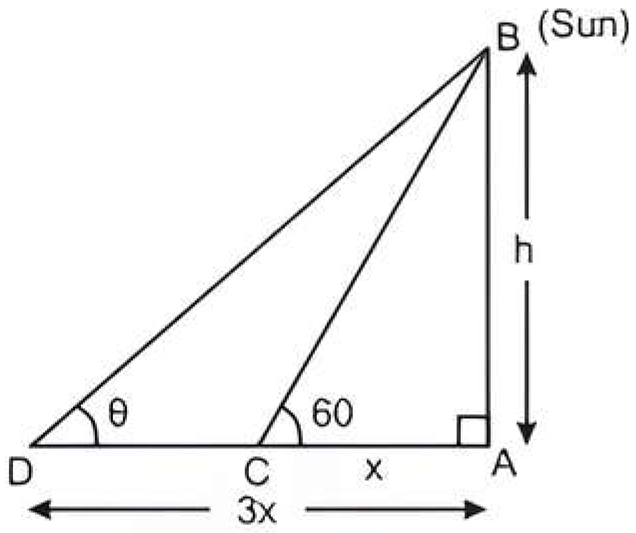
$$= 86.50 + 1.7$$

$$\Rightarrow \quad h = 88.20 \text{ m}$$

Hence, height of the building is 88.20 m.

2. Suppose B be the position of the Sun. Let the height of the tower be h m and the angle between the Sun and the ground at the time of longer shadow be θ.

AC and AD are the lengths of the shadow of the tower when the angle between the Sun and the ground are 60° and θ, respectively.



Let $AC = x$ unit, then

Given, $AD = 3\,AC \Rightarrow AD = 3x$

In right-angled ΔBAC,

$$\tan 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = x\sqrt{3} \quad \text{...(1)}$$

In right-angled ΔBAD,

$$\tan\theta = \frac{AB}{AD} = \frac{h}{3x}$$

$$\Rightarrow \quad h = 3x \times \tan\theta \quad \text{...(2)}$$

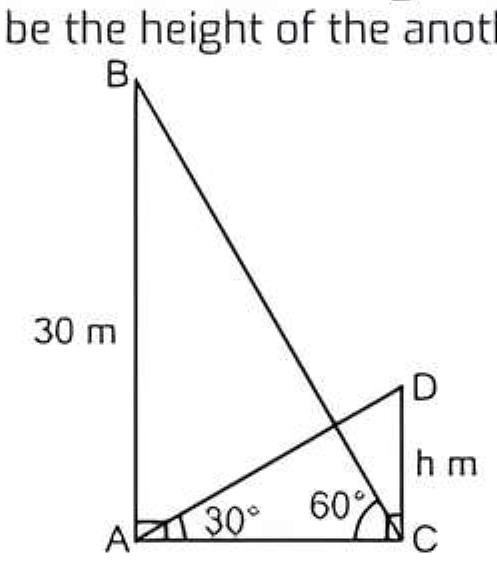
From eqs. (1) and (2), we get

$$x\sqrt{3} = 3x \cdot \tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

3. Let $AB = 30$ m be the height of the tower and $CD = h$ m be the height of the another tower. Then



$\angle CAD = 30^\circ$ and $\angle ACB = 60^\circ$.

In right-angled ΔACB,

$$\tan 60^\circ = \frac{AB}{AC} \quad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \sqrt{3} = \frac{30}{AC} \Rightarrow AC = \frac{30}{\sqrt{3}} \quad \text{...(1)}$$

$$\Rightarrow \quad AC = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\text{ m}$$

In right-angled ΔCAD,

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{10\sqrt{3}} \quad [\because \text{from eq. (1)}]$$

$$\Rightarrow \quad h = \frac{10\sqrt{3}}{\sqrt{3}} = 10 \text{ m}$$

Hence, the distance between the two towers and height of the other tower are $10\sqrt{3}$ m and 10 m respectively.

4. Let $AB = h$ m be the height of the tower.

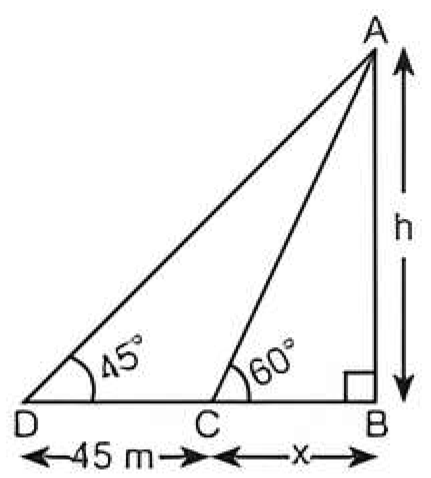
Let $BC = x$ m and $DC = 45$ m

In right-angled ΔABC,

$$\tan 60^\circ = \frac{AB}{BC} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3}x \qquad \text{...(1)}$$



In right-angled ΔABD,

$$\tan 45^\circ = \frac{AB}{BD} = \frac{AB}{BC + CD}$$

$$\Rightarrow 1 = \frac{h}{x + 45}$$

$$\Rightarrow h = x + 45$$

$$\Rightarrow h = \frac{h}{\sqrt{3}} + 45 \qquad \text{(using eq. (1))}$$

$$\Rightarrow h - \frac{h}{\sqrt{3}} = 45$$

$$\Rightarrow \frac{(\sqrt{3}-1)h}{\sqrt{3}} = 45$$

$$\Rightarrow h = \frac{45\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

(rationalising the denominator)

$$= \frac{45(3+\sqrt{3})}{(\sqrt{3})^2 - 1} \qquad (\because (a-b)(a+b) = a^2 - b^2)$$

$$= \frac{45(3 + 1.732)}{3-1}$$

$$= \frac{45 \times 4.732}{2} = 106.47 \text{ m}$$

Hence, the height of the tower is 106.47 m.

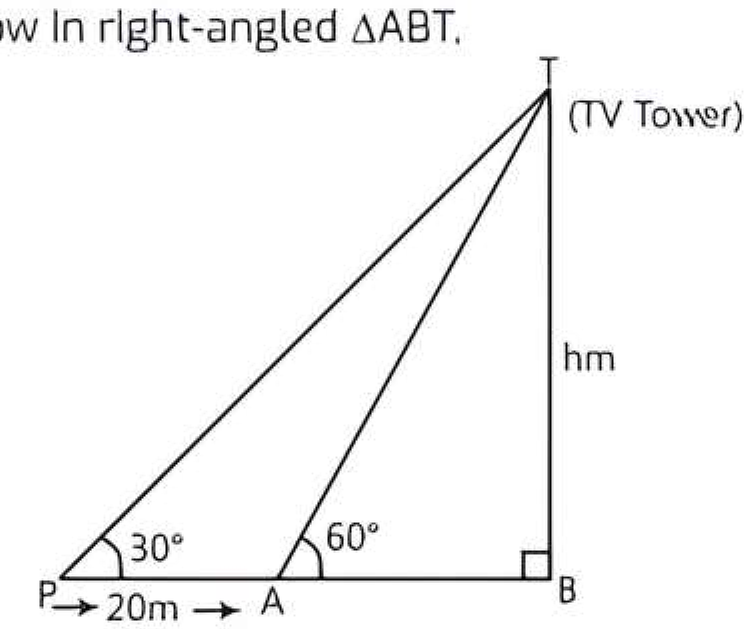
COMMON ERR(!)R

Sometime students get confused with the values of trigonometric angles. They substitute wrong values which leads to the wrong result.

5. Let a TV tower (TB) stands vertically on the bank (B) of a canal. From a point A on the other bank directly opposite the tower, the angle of elevation of the top T of the tower is $\angle TAB = 60^\circ$.

From another point P, AP = 20m away from the point A on the line joining this point to the foot (B) of the tower, the angle of elevation of the top T of the tower is $\angle TPB = 30^\circ$. Suppose height of the tower TB = hm.

Now In right-angled ΔABT,



$$\tan 60^\circ = \frac{TB}{AB} = \frac{h}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{AB} \Rightarrow AB = \frac{h}{\sqrt{3}} \qquad \text{...(1)}$$

In right-angled ΔPBT,

$$\tan 30^\circ = \frac{TB}{PB} = \frac{h}{PA + AB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20 + \frac{h}{\sqrt{3}}} \qquad \text{[from eq. (1)]}$$

$$\Rightarrow 20 + \frac{h}{\sqrt{3}} = h\sqrt{3}$$

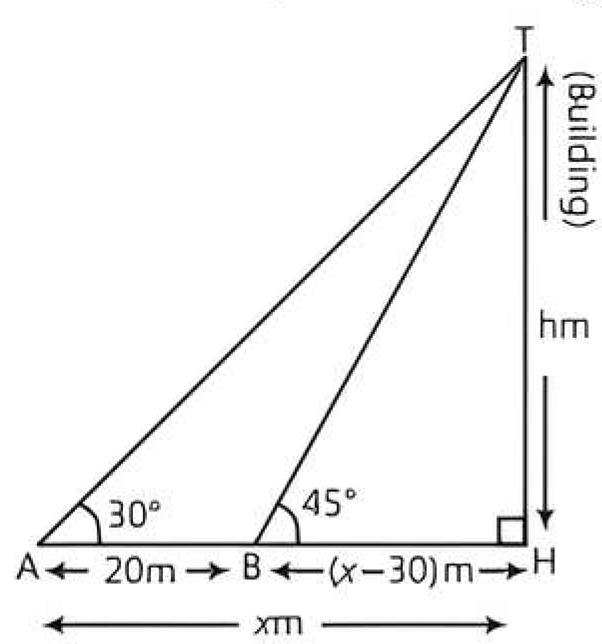
$$\Rightarrow h\sqrt{3} - \frac{h}{\sqrt{3}} = 20$$

$$\Rightarrow \frac{3h - h}{\sqrt{3}} = 20 \Rightarrow 2h = 20\sqrt{3}$$

$$\Rightarrow h = 10\sqrt{3}$$

Hence, the height of the tower is $10\sqrt{3}$m.

6. Let the angle of elevation of the top T of a building (TH) from a point A on the ground is $\angle TAH = 30^\circ$. On moving a distance AB = 30m towards its base to the point B, the angle of elevation changes to $\angle TBH = 45^\circ$.



Suppose the height TH = hm of the building and the distance of its base from point A is AH = xm.

Now In right-angled ΔBHT,

$$\tan 45^\circ = \frac{TH}{BH} = \frac{h}{x - 30}$$

$$\Rightarrow 1 = \frac{h}{x-30} \Rightarrow x - 30 = h$$

$$\Rightarrow x = h + 30 \qquad \text{...(1)}$$

In right-angled ΔAHT,

$$\tan 30^\circ = \frac{TH}{AH} = \frac{h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = h\sqrt{3} \quad ...(2)$$

From eqs. (1) and (2), we get

$$h + 30 = h\sqrt{3}$$

$$\Rightarrow \sqrt{3}h - h = 30$$

$$\Rightarrow (\sqrt{3} - 1)\, h = 30$$

$$\Rightarrow (1.732 - 1)\, h = 30$$

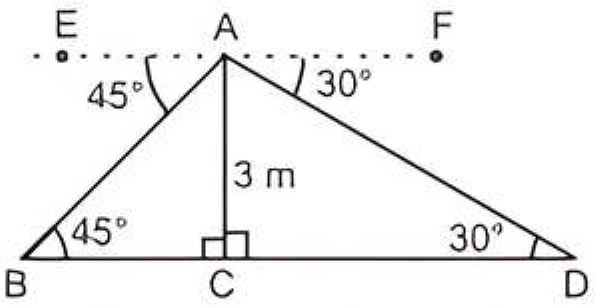
$$\Rightarrow h = \frac{30}{0.732} = 40.98\text{m}$$

From (1),

$x = 40.98 + 30 = 70.98$m

Hence, height of the building and the distance of its base from point A are 40.98m and 70.98m respectively.

7. Let A be the point of the bridge and B and D be the position of the opposite sides of the bridge.



Also, given the angle of depressions are

$\angle EAB = 45°$ and $\angle FAD = 30°$

Then,

$\angle ABC = \angle EAB = 45°$ (by alternate angles)

and $\angle ADC = \angle FAD = 30°$ (by alternate angles)

In right-angled ΔBCA,

$$\tan 45° = \frac{AC}{BC}$$

$$\Rightarrow 1 = \frac{AC}{BC} \Rightarrow BC = 3\,\text{m}$$

In right-angled ΔDCA,

$$\tan 30° = \frac{AC}{CD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{CD} \Rightarrow CD = 3\sqrt{3}\ \text{cm}$$

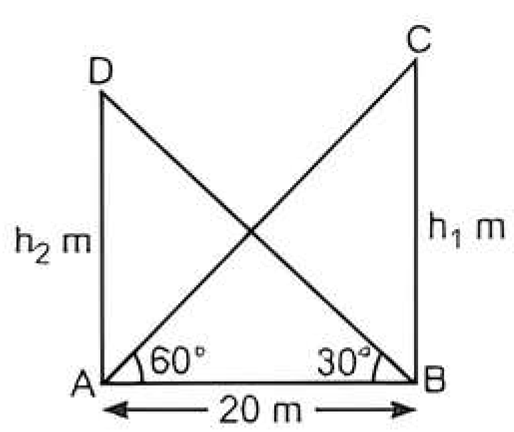
$$BD = BC + CD$$

$$= 3 + 3\sqrt{3}$$

$$= 3(1+\sqrt{3})\,\text{m}$$

Hence, width of the river is $3(1+\sqrt{3})$ m.

8. Let heights of two different poles be BC = h_1 m and AD = h_2 m.



Also given, $\angle BAC = 60°$

$\angle ABD = 30°$ and OAB = 20 m

In right-angled Δ ABC,

$$\tan 60° = \frac{BC}{AB} \quad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \sqrt{3} = \frac{h_1}{20}$$

$$\Rightarrow h_1 = 20\sqrt{3}\ \text{m}$$

In right-angled ΔBAD,

$$\tan 30° = \frac{AD}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h_2}{20}$$

$$\Rightarrow h_2 = \frac{20}{\sqrt{3}} = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{20\sqrt{3}}{3}\text{m}$$

$\therefore$ The difference between the height of two poles

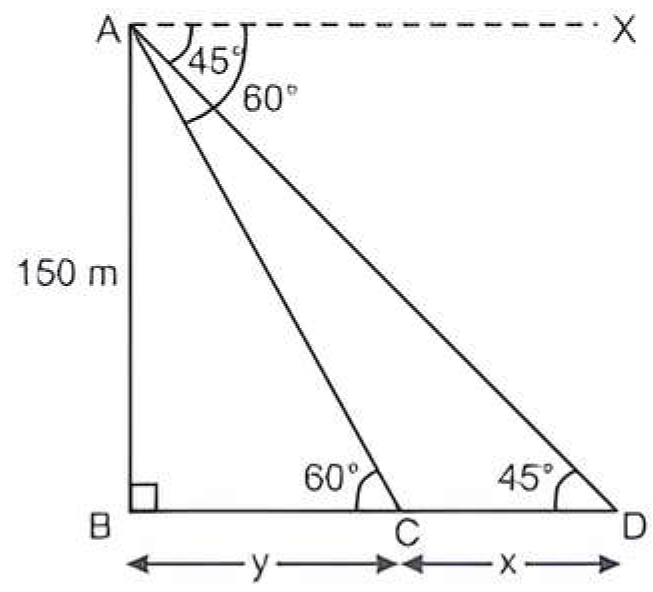
$$= h_1 - h_2 = 20\sqrt{3} - \frac{20}{3}\sqrt{3}$$

$$= \frac{60\sqrt{3} - 20\sqrt{3}}{3} = \frac{40\sqrt{3}}{3}$$

$$= \frac{40 \times 1.73}{3} = \frac{69.2}{3}$$

$$= 23.07\ \text{m}$$

9. In the figure, AB represents the 150 m high cliff. Initially, the boat is at point C and it moves to point D in 2 min and as it is given that the angle of depression of the boat changes from 60° to 45°.



So, $\angle DAX = 45°$ and $\angle CAX = 60°$

TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

$\therefore$ $\angle DAX = \angle ADB = 45°$ and $\angle CAX = \angle ACB = 60°$ (alternate interior angles)

In right-angled ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{150}{y} \quad (\because AB = 150\ \text{m and } BC = y\ \text{m})$$

$\Rightarrow \quad y = \frac{150}{\sqrt{3}} = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$ m

In right-angled ΔABD,

$$\tan 45° = \frac{AB}{BD} = \frac{AB}{BC + CD}$$

$\Rightarrow \quad 1 = \frac{150}{y + x} \qquad (\because y = 50\sqrt{3}\text{ m})$

$\Rightarrow \quad x + y = 150$

$\Rightarrow \quad x = 150 - 50\sqrt{3}$ m

∵ The boat covers CD distance in 2 min.

∴ The speed of boat $= \frac{\text{distance}}{\text{time}} = \frac{CD}{2}$ m/min

$= \frac{x}{(2)}$ m/min

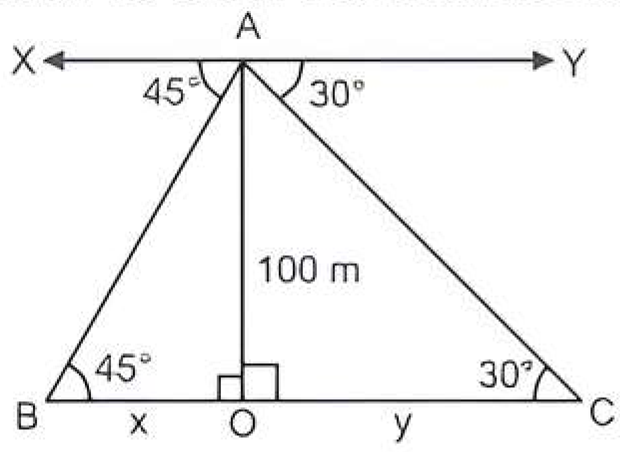
$= \frac{1}{2}(150 - 50\sqrt{3})$ m/min

$= 25(3 - \sqrt{3})$ m/min

$= 25\sqrt{3}(\sqrt{3} - 1)$ m/min

Long Answer Type Questions

1. Let the top of the tower AO, a man at point A, observed the angle of depression 30° of car C and angle of depression 45° of car B on both sides of a tower.



TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

∴ ∠XAB = ∠ABO = 45°

and ∠YAC = ∠ACO = 30° (alternate angles)

Given, OA = 100 m

TiP

The concept of angle of depression must be understood deeply and clearly.

In right-angled ΔAOB,

$\tan 45° = \frac{OA}{OB} = \frac{100}{x}$ (let OB = x m)

$\Rightarrow \quad 1 = \frac{100}{x} \Rightarrow x = 100$ m ...(1)

In right-angled ΔAOC,

$\tan 30° = \frac{OA}{OC} = \frac{100}{y}$ (let OC = y m)

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{y}$

$\Rightarrow \quad y = 100\sqrt{3}$ m ...(2)

Therefore, width of the river = x + y

$= 100 + 100\sqrt{3}$

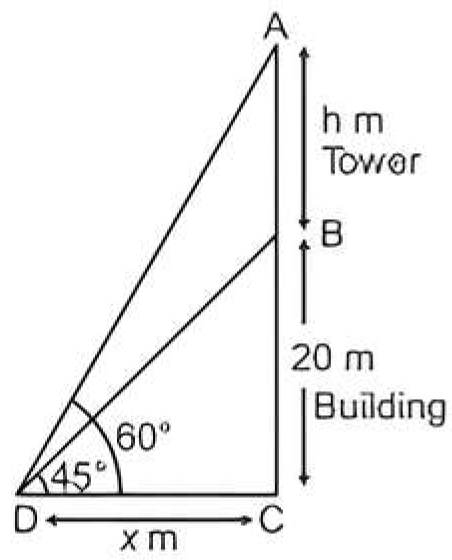
$= 100 + 100 \times 1.73 = 100 + 173$

$= 273$m

Hence, distance between two cars is 273 m.

2. Let AB be the tower and BC be the building.

Let DC = x m, AB = h m and BC = 20 m.



In right-angled ΔBCD,

$\tan 45° = \frac{BC}{DC} \Rightarrow 1 = \frac{20}{x}$

$\Rightarrow \quad x = 20$ m

In right-angled ΔACD,

$\tan 60° = \frac{AC}{DC} = \frac{AB + BC}{DC}$

$\Rightarrow \quad \sqrt{3} = \frac{h + 20}{x}$

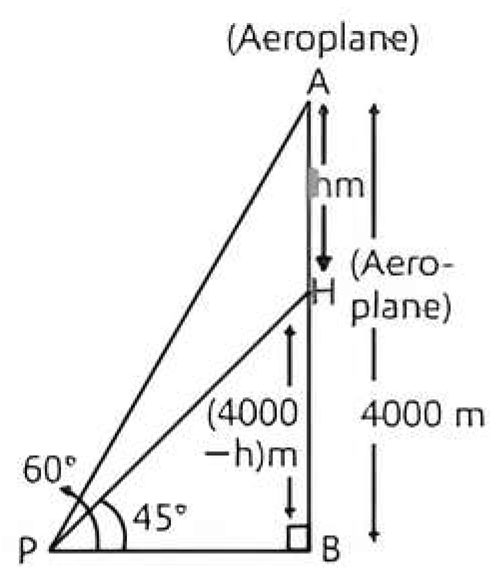
$\Rightarrow \quad \sqrt{3}x = h + 20 \qquad \Rightarrow \sqrt{3} \times 20 = h + 20$

$(\because x = 20\text{ m})$

$\Rightarrow \quad h = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$ m

Hence, the height of tower is $20(\sqrt{3} - 1)$ m.

3. Suppose an aeroplane (A) when flying at a height AB = 4000 m from the ground passes vertically above another aeroplane(H) at an instant when the angles of elevation of the two planes from the same point (P) on the ground are



∠APB = 60° and ∠HPB = 45° respectively.

Let the vertical distance between the aeroplanes at that instant (AH) = hm.

∴ HB = AB − AH = (4000 − h) m.

In right-angled ΔPBH.

$\tan 45° = \frac{HB}{PB} \Rightarrow 1 = \frac{4000 - h}{PB}$

$\Rightarrow$ PB = 4000 − h ...(1)

In right-angled ΔPBA.

$\tan 60° = \frac{AB}{PB} \Rightarrow \sqrt{3} = \frac{4000}{PB}$

$\Rightarrow \sqrt{3}\,PB = 4000 \Rightarrow \sqrt{3}(4000 - h) = 4000$

[from eq. (1)]

$\Rightarrow \quad h = 4000 - \frac{4000}{\sqrt{3}} = 4000 - \frac{4000}{1.732}$

$= 4000 - 2309.47$

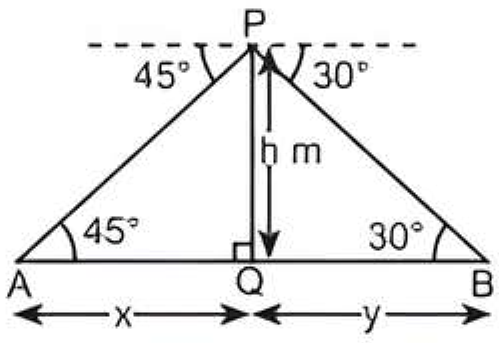
$= 1690.53$ m.

Thus, the vertical distance between the aeroplanes at that instant is 1690.53 m.

4. Let PQ be the lighthouse, and A and B are the position of two ships.

TiP

The concept of angle of depression must be understood deeply and clearly.



Let $PQ = h$ m, $AQ = x$ m and $QB = y$ m.

The distance between two ships (AB) = $x + y = 100$ m (Given)

In right-angled Δ PQA,

$$\tan 45^\circ = \frac{PQ}{AQ} \Rightarrow 1 = \frac{h}{x} \Rightarrow x = h \quad ...(1)$$

In right-angled Δ PQB,

$$\tan 30^\circ = \frac{PQ}{QB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{y}$$

$$\Rightarrow \quad y = \sqrt{3}\,h \quad ...(2)$$

Adding eqs. (1) and (2), we get

$$x + y = h + \sqrt{3}\,h$$

$$\Rightarrow \quad h + \sqrt{3}h = 100 \quad [\because x + y = 100 \text{ m}]$$

$$\Rightarrow \quad (\sqrt{3} + 1)\,h = 100$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

(rationalising the denominator)

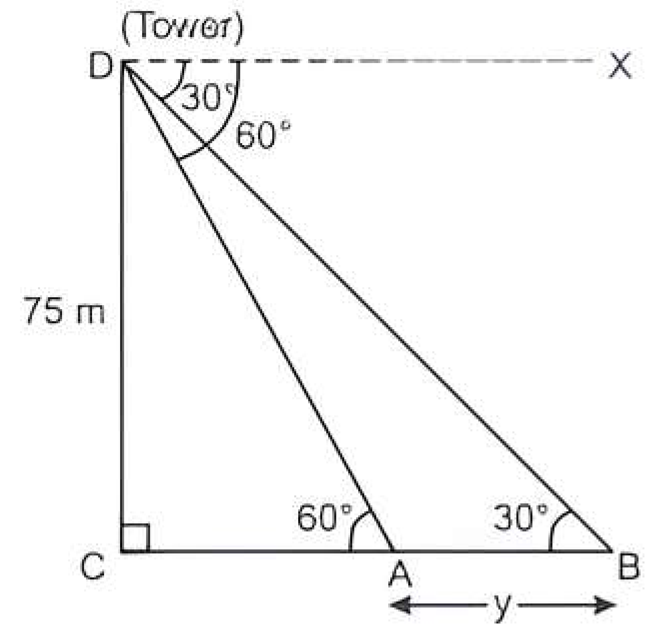
$$= \frac{100\,(\sqrt{3}-1)}{3-1} = \frac{100\,(1.732-1)}{2}$$

$$(\because (a+b)\,(a-b) = a^2 - b^2)$$

$$= 50 \times 0.732 = 36.6 \text{ m}$$

Hence, the height of the lighthouse is 36.6 m.

5. Given, the angles of depression of two cars A and B from the man standing on the top D of the tower CD (say) with height 75 m are 60° and 30° respectively.



TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

$\therefore \quad \angle XDB = \angle CBD = 30^\circ$

and $\quad \angle XDA = \angle CAD = 60^\circ$ (alternate angles)

Let the distance between the cars, $AB = y$ m

In right-angled ΔACD,

$$\tan 60^\circ = \frac{CD}{CA} \Rightarrow \sqrt{3} = \frac{75}{CA}$$

$$\Rightarrow \quad CA = \frac{75}{\sqrt{3}} \quad ...(1)$$

In right-angled ΔBCD,

$$\tan 30^\circ = \frac{CD}{BC} = \frac{CD}{CA + AB} = \frac{75}{CA + y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{\frac{75}{\sqrt{3}} + y} \quad [\text{from eq. (1)}]$$

$$\Rightarrow \quad y = 75\sqrt{3} - \frac{75}{\sqrt{3}} = \frac{75}{\sqrt{3}}(3-1) = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{150\sqrt{3}}{3} = 50\sqrt{3} = 50 \times 1.73 = 86.5$$

So, required distance between two cars is 86.5m.

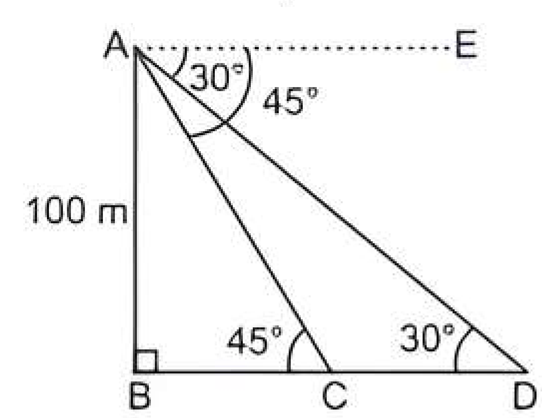
6. Let AB be the light house, C and D be the position of the ships.

TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

$\therefore \quad \angle EAD = \angle ADB = 30^\circ$

and $\angle EAC = \angle ACB = 45^\circ$ (alternate interior angles)



In right-angled ΔABC,

$$\tan 45^\circ = \frac{AB}{BC} = \frac{100}{BC}$$

$$\Rightarrow \quad 1 = \frac{100}{BC} \quad \Rightarrow BC = 100 \text{ m}$$

In right-angled ΔABD,

$$\tan 30^\circ = \frac{AB}{BD} = \frac{100}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{BD} \Rightarrow BD = 100\sqrt{3}\,m$$

$\therefore$ The distance between the two ships,

$$CD = BD - BC = 100\sqrt{3} - 100$$
$$= 100(\sqrt{3}-1) = 100(1.732-1)$$
$$= 100 \times 0.732 = 73.2\,m$$

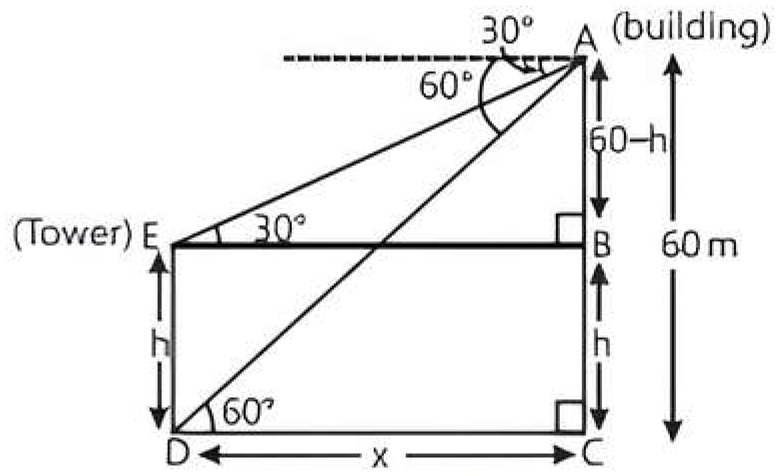
Hence, the distance between the two ships is 73.2 m.

7. In the figure, DE represents the tower and AC represent 60 m high building. The distance between building and tower is DC = xm.

Let the height of the tower be h m.

TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.



$\therefore \quad \angle XAE = \angle AEB = 30^\circ$

and $\angle XAD = \angle ADC = 60^\circ$ (alternate interior angles)

In right-angled ΔABE,

$$\tan 30^\circ = \frac{AB}{BE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{x}$$

$(\because DE = BC = h \therefore AB = AC - BC = AC - DE = 60 - h)$

$$\Rightarrow \quad x = \sqrt{3}\,(60-h) \qquad ...(1)$$

In right-angled ΔACD,

$$\tan 60^\circ = \frac{AC}{DC} = \frac{AC}{x} \qquad (\because BE = DC = xm)$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{\sqrt{3}\,(60-h)} \qquad \text{[from eq. (1)]}$$

$$\Rightarrow \quad 60 = 3\,(60-h)$$

$$\Rightarrow \quad 20 = 60 - h$$

$$\Rightarrow \quad h = 40\text{ m}$$

$$x = \sqrt{3}\,(60-h) \qquad [\because \text{from eq. (1)}]$$

$$x = \sqrt{3}\,(60-40) = 20\sqrt{3} = 20 \times 1.732$$
$$= 34.64\text{ m}$$

Hence, distance between the building and the tower is 34.64 m and height of the tower is 40 m.

COMMON ERR(!)R

The concept of angle of depression and angle of elevation are not clear to many students. That's why they are not able to draw the diagram correctly.

Chapter Test

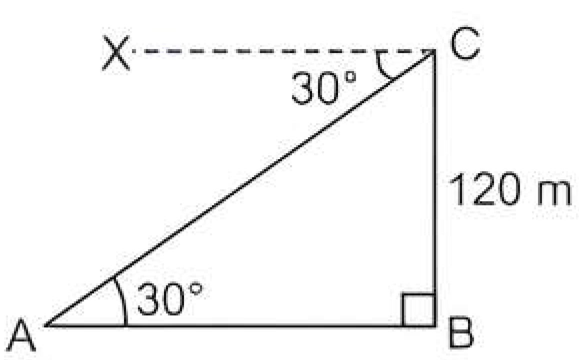
Multiple Choice Questions

Q 1. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then Sun's elevation is:

a. 60°   b. 45°

c. 30°   d. 90°

Q 2. The angle of depression of a car parked on the road from the top of 120 m high tower is 30°. The distance of the car from the tower (in metres) is:



a. $120\sqrt{3}$ m   b. 120 m

c. $40\sqrt{3}$ m   d. None of these

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct choices as.*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°.

Reason (R): Trigonometric ratio, tangent is defined as $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

Q 4. Assertion (A): The angle of elevation of the top of the tower is 30° and the horizontal distance from the observer's eye to the foot of the tower is 50 m, then the height of the tower will be $\frac{50}{3}\sqrt{3}$ m.

Reason (R): While using the concept of angle of elevation/depression, triangle should be a right-angled triangle.

Fill in the Blanks

Q 5. The .... of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Q 6. The angle of .... of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level.

True/False

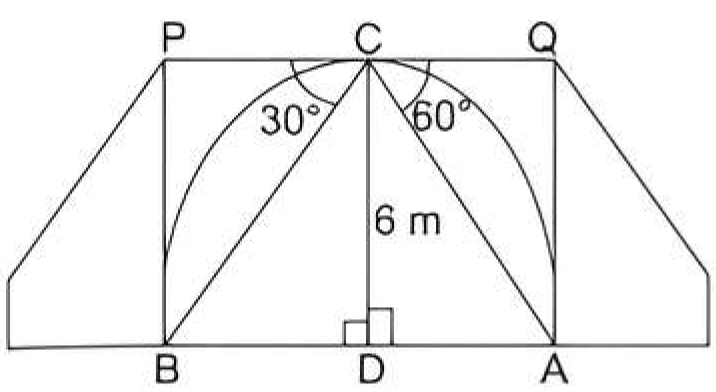
Q 7. The height of the tower if length of the shadow is 10 m and Sun's altitude is 45°, is 10 m.

Q 8. If length of shadow of tower is 20 m and angle of elevation is 60°, then the height of tower is $\frac{20}{\sqrt{3}}$ m.

Case Study Based Question

Q 9. One day while sitting on the bridge across a river Ankit observes the angles of depression of the banks on opposite sides of the river are 30° and 60° respectively as shown in the figure.

(Take √• = ••••)



Based on the above information, solve the following questions:

(i) If the bridge is at a height of 6 m, then , find the length of AD.

(ii) Find the length of BD.

(iii) Find the width of the river.

Or

Find the distance of Ankit (c) from both banks of the river.

Very Short Answer Type Questions

Q 10. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall.

Q 11. A kite is flying, attached to a thread which is 140 m long. The thread makes an angle of 30° with the ground. Find the height of the kite from the ground, assuming that there is no slack in the thread.

Short Answer Type-I Questions

Q 12. The length of a string between a kite and a point on the ground is 70 m. If the string makes an angle θ with the ground level such that $\tan\theta = \frac{4}{3}$ then the kite is at what height from the ground?

Q 13. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Short Answer Type-II Questions

Q 14. A person walking 45 m towards a tower in a horizontal line through its base observes that angle of elevation of the top of the tower changes from 45° to 60°. Find the height of the tower.

Q 15. From a point O on the ground, the angle of elevation of the top of a tower is 30 and that of the flagstaff on the top of the tower is 60°. If the length of the flagstaff is 5 m, find the height of the tower.

Long Answer Type Question

Q 16. Two ships are approaching a lighthouse from opposite directions. The angles of depression of the two ships from the top of a lighthouse are 30° and 45°. If the distance between the two ships is 100 m, find the height of the lighthouse.

[Use $\sqrt{3} = 1.732$]